Mathematics 300

Quiz 31 Name: Key

You must show your work to get full credit.

1. Let $F_1, F_2, F_3, \ldots$ be the Fibonacci numbers. Prove that $F_2 + F_4 + F_6 + \cdots F_{2n} = F_{2n+1} - 1$.

Recall the first several Fibonacci numbers are
$F_1 = 1, F_2 = 1, F_3 = 2, \ldots$

Base case: $n=1$ $F_2 = 1 = 2-1 = F_3 - 1$.

Induction hypothesis: $F_2 + F_4 + \cdots + F_{2k} = F_{2k+1} - 1$

Add $F_{2(k+1)} = F_{2k+2}$ to both sides:

$$\begin{aligned} F_2 + F_4 + \cdots + F_{2k} + F_{2k+2} &= F_{2k+1} - 1 + F_{2k+2} \\ &= (F_{2k+1} + F_{2k+2}) - 1 \\ &= F_{2k+3} - 1 \\ &= F_{2(k+1)+1} - 1 \end{aligned}$$

which finishes the induction.

2. Use strong induction to show that an positive integers $n \geq 2$ is a product of primes.

Base case: $n=2$. Then 2 is prime, so we are done.

Induction hypothesis: If $2 \le l \le k$, then l is a product of primes.

Then consider $k+1$.

Case 1 $k+1$ is prime. Then we are done

Case 2 $k+1$ is not prime. Then $k+1 = ab$ where a, b are integers, with $2 \le a \le k$, $2 \le b \le k$. By the induction hypothesis a and b are products of primes, say $a = p_1 \cdots p_s$, $b = q_1 \cdots q_t$. Then $ab = k+1 = p_1 \cdots p_s q_1 \cdots q_t$ is a product of primes. This finishes the induction.