Mathematics 300

Quiz 32 **Name:**_____

You must show your work to get full credit.

Let

$$f(x) = (x+1)e^x.$$

1. Compute the first four derivatives of $f(x)$.

$$f'(x) = (x+2)e^x$$

$$f''(x) = (x+3)e^x$$

$$f^{(3)}(x) = (x+4)e^x$$

$$f^{(4)}(x) = (x+5)e^x$$

2. Make conjecture about $f^{(n)}(x)$. $f^{(n)}(x) = (x+n+1)e^x$ _____

3. Prove your conjecture. We use proof by induction.

Base case: $n = 1$. Then the conjecture is $f'(x) = (x+1+1)e^x = (x+2)e^x$ which we have seen is true.

Induction hypothesis: $f^{(k)} = (x+k+1)e^x$.

Assume this holds we compute $f^{(k+1)}$.

$$\begin{aligned} f^{(k+1)}(x) &= (f^{(k)}(x))' \\ &= ((x+k+1)e^x)' \\ &= (x+k+1)'e^x + (x+k+1)(e^x)' \\ &= 1e^x + (x+k+1)e^x \\ &= (1+x+k+1)e^x \\ &= (x+(k+1)+1)e^x \end{aligned}$$

which is the **induction conclusion**. This completes the induction and the proof.