Mathematics 300

Quiz 33 Name: key

You must show your work to get full credit.

1. Let a, r be real numbers with $r > 0$ and let $n \geq 0$ be in integer. Prove that

$$a + ar + \cdots + ar^n = \frac{1 - ar^{n+1}}{1 - r} = \frac{\text{first} - \text{next}}{1 - \text{ratio}}$$

Let $S = a + ar + \cdots + ar^n$

Then $rS = ar + \cdots + ar^n + ar^{n+1}$

Subtract $S - rS = a - ar^{n+1}$

$S(1-r) = a - ar^{n+1}$

$S = \frac{1 - ar^{n+1}}{1-r}$

2. Find the sum $6 + 12 + 3 \cdot 2^3 + \cdots 3 \cdot 2^{10}$ Sum = $3 \cdot 2^{11} - 6 = 6138$

$S = \frac{\text{first} - \text{next}}{1 - \text{ratio}}$

$= \frac{6 - 3 \cdot 2^{11}}{1 - 2} = \frac{6 - 3 \cdot 2^{11}}{-1} = 3 \cdot 2^{11} - 6$

3. Prove that

$$a + (a + d) + (a + 2d) + \cdots + (a + (n-1)d) = n\left(\frac{a + (a + (n-1)d)}{2}\right)$$

$$= (\text{number of terms})\left(\frac{\text{first} + \text{last}}{2}\right)$$

Let $S = a + (a+d) + \cdots + (a + (n-1)d)$

so $S = (a + (n-1)d) + \cdots + a$

$2S = S + S = \underbrace{[a + (a + (n-1)d)] + \cdots + [a + (n-1)d + a]}_{n \text{ terms}}$

$= n(a + (a + (n-1)d))$

so $S = \frac{n(a + (a + (n-1)d))}{2}$

4. Find the sum of $100 + 102 + 104 + \cdots + 200$. Sum = 7650

There are 51 terms so

Sum $= 51\left(\frac{100 + 200}{2}\right) = 51(150) = 7650$