Mathematics 300

Quiz 35 **Name:**________________________________

You must show your work to get full credit.

1. Use induction to show for any real numbers $a, b_1, b_2, \ldots, b_n$ that

$$a(b_1 + b_2 + \cdots + b_n) = a_1 b + ab_2 + \cdots + ab_n.$$

(You may assume that $a(b + c) = ab + ac$ holds.)

Proof. **Base case:** For $n = 1$ the identity becomes $ab_1 = ab_1$ with is true. **Induction hypothesis:** $a(b_1 + b_2 + \cdots + b_k) = a_1 + ab_2 + \cdots + ab_k$.

Now

$$\begin{aligned} a(b_1 + b_2 + \cdots b_k + b_{k+1}) &= a(b + c) && \text{(where } b = b_1 + \cdots + b_k \text{ and } c = b_{k+1}\text{)} \\ &= ab + ac \\ &= a(b_1 + b_2 + \cdots + b_k) + ab_{k+1} \\ &= a_1 + ab_2 + \cdots + ab_k + ab_{k+1} && \text{(using the induction hypothesis).} \end{aligned}$$

This shows the induction conclusion holds and completes the proof. □

2. Use induction to show for any sets A, B_1, B_2, $\ldots$, B_n that

$$A \cup (B_1 \cap B_2 \cap \cdots \cap B_n) = (A \cup B_1) \cap (A \cup B_2) \cap \cdots \cap (A \cup B_n).$$

(You may assume that $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$ holds.)

Proof. **Base case:** For $n = 1$ this just $A \cup B_1 = A \cap B_2$ which is true.

Induction hypothesis: $A \cup (B_1 \cap B_2 \cap \cdots \cap B_k) = (A \cup B_1) \cap (A \cup B_2) \cap \cdots \cap (A \cup B_k)$.

Then, using the notation $B = B_1 \cap B_2 \cap \cdots \cap B_k$ and $C = B_{k+1}$,

$$\begin{aligned} A \cup (B_1 \cap B_2 \cap \cdots \cap B_{k+1}) &= A \cup (B \cap C) \\ &= (A \cup B) \cap (A \cup C) \\ &= \Big(A \cup (B_1 \cap B_2 \cap \cdots \cap B_k)\Big) \cup A \cap B_{k+1} \\ &= (A \cup B_1) \cap \cdots \cap (A \cup B_k) \cup (A \cap B_{k+1}) && \text{(by the ind. hyp.).} \end{aligned}$$

This finishes the induction. □