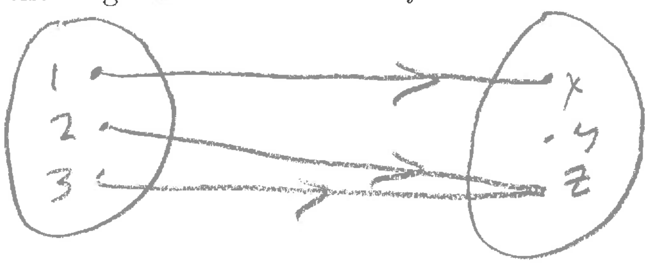
Mathematics 300

Quiz 36 Name: Key

You must show your work to get full credit.

1. Let $A = \{1, 2, 3\}$ and $B = \{x, y, z\}$ and $f = \{(1, x), (2, z), (3, z)\}$.
(a) Draw the diagram for the function f.



(b) What is $f(2)$? $f(2) =$ z

(c) Is f injective? ($2 \neq 3$, but $f(2) = f(3)$) Yes or no? NO

(d) Is f surjective (y is not in the range.) Yes or no? NO.

2. Show the function $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x + 3$ is both injective and surjective.
(a) Proof that f is surjective:

To show that f is surjective we need to show that for all $b \in \mathbb{R}$, there is a solution to $f(a) = b$ with $a \in \mathbb{R}$.

$$f(a) = 2a + 3 = b$$
$$2a = b - 3$$
$$a = \frac{b-3}{2} \in \mathbb{R}$$

Thus f is surjective

(b) Proof that f is injective:

To show f is injective we need to show that if $f(a) = f(b)$, then $a = b$.

$$f(a) = 2a + 3 = 2b + 3 = f(b)$$
$$2a = 2b \quad \text{(subtract 3)}$$
$$a = b \quad \text{(divide by 2)}$$

so f is injective.

3. Show that the function $h\colon \mathbb{Z} \to \mathbb{Z}$ given by $h(n) = 3n + 1$ is injective, but not surjective.

(a) Proof that h is injective.

To show h is injective we need to show that if $h(a) = h(b)$, then $a = b$

$$h(a) = 3a + 1 = 3b + 1 = h(b)$$

$$3a = 3b \quad \text{(substract 1)}$$

$$a = b \quad \text{(divide by 3)}$$

Thus h is injective.

(b) Proof that h is not surjective.

To see if h is surjective we need to check if for all $b \in \mathbb{Z}$ we can solve $h(a) = b$ for a with $a \in \mathbb{Z}$. $h(a) = 3a + 1 = b$

$$3a = b - 1$$

$$a = \frac{b-1}{3}$$

so if $b = 2$, $a = \frac{2-1}{3} = \frac{1}{3} \notin \mathbb{Z}$. Thus h is not surjective

4. Let $g\colon \mathbb{R} \to \mathbb{R}$ be the function $g(x) = 5x - 4$ is bijective and find the inverse of g.

(a) Proof that g is injective. We need to show if $g(a) = g(b)$, then $a = b$.

$$5a - 4 = 5b - 4$$

$$5a = 5b \quad \text{(add 4)}$$

$$a = b \quad \text{(divide by 5)}$$

so g is injective.

(b) Proof that g is surjective.

Let $b \in \mathbb{R}$ we need to show that there is an $a \in \mathbb{R}$ with $g(a) = b$. $g(a) = 5a - 4 = b$

$$5a = b + 4$$

$$a = \frac{b+4}{5} \in \mathbb{R}$$

so g is surjective.

(c) Inverse of g. Using the calculation $g^{-1}(x) = \underline{\frac{x+4}{5}}$.
of (b) we see $g^{-1}(b) = \frac{b+4}{5}$

Now replace b by x