Mathematics 300 Homework, September 22, 2017.

Read Section 4.1, 4.2 and 4.3 pages 88–98. You should memorize the definitions of ***even***, ***odd***, ***same parity***, ***opposite parity*** (which I assume you already know) and a ***divides*** b (in symbols $a \mid b$), a is a ***factor*** of b, a is a divisor of b, and p is a ***prime***.

On pages 100 and 101 do problems 7, 11, 15, and 19.

Here are a couple of other problems.

1. If a, b, and d are integers, and $d \mid a$ and $d \mid b$, then $d \mid (2a - 3b)$.

2. If a and b are integers and $a \mid b$, then $3a^2 \mid (6ab - 12a^3)$.

3. If a, b and c are integers and $a \mid b$ and $b \mid c$, then $a^2 \mid bc$.

4. If a, b, d are integers and $d \mid a$ and $d \mid b$, then for any integers x and y we have $d \mid (ax + by)$.

The solutions for these start on the next page.

Solution for Problem 1. As $d \mid a$ and $d \mid b$, by definition there are integers m and n such that

$$a = md$$
$$b = nd.$$

Therefore

$$\begin{aligned} 2a - 3b &= 2dm - 3dn \\ &= (2m - 3n)d \\ &= cd \end{aligned}$$

where $c = 2m - 3n$ is an integer. Thus $d \mid (2a - 3b)$. □

Solution for Problem 2. By the definition of $a \mid b$ we have

$$b = am$$

for some integer m. Therefore

$$\begin{aligned} 6ab - 12a^3 &= 6a(am) - 12a^3 \\ &= 3a^2(2m - 4a) \\ &= 3a^2k \end{aligned}$$

where $k = 2m - 4a$ is an integer. Thus $3a^2 \mid (6ab - 12a^2)$. □

Solution for Problem 3. As $a \mid b$ and $b \mid c$

$$b = am$$
$$c = bn$$

for some $m, n \in \mathbb{Z}$. Therefore

$$\begin{aligned} bc &= (am)(bn) \\ &= (am)((am)n) && \text{(Where we have used } b = am \text{ again)} \\ &= m^2na^2 \\ &= ka^2 \end{aligned}$$

where $k = m^2n$ is an integer. Whence $a^2 \mid bc$. □

Solution for Problem 4. Because $d \mid a$ and $d \mid b$

$$a = md$$
$$b = nd$$

for some integers $m, n \in \mathbb{Z}$. Then if x and y are integers

$$\begin{aligned} ax + by &= (md)x + (nd)y \\ &= (mx + ny)d \\ &= kd \end{aligned}$$

where $k = mx + ny$ is an integer. Thus $d \mid (ax + by)$.

□