

## Mathematics 300 Homework, October 11, 2017.

Read Chapter 6, Pages 111–117. He does a good job of explaining how proof by contradiction works and has some nice examples.

For homework do Problems 1 (which I think we have done before), 3 5 (for this one you can use the fact we proved in class today, then if  $a$  is an integer and  $3 \mid a^2$ , then  $3 \mid a$ ), 7, 9.

Here are two problems to be turned in.

1. The number  $\frac{1 + \sqrt{2}}{1 - \sqrt{2}}$  is irrational.

2. Let  $n$  be an integer with  $n \geq 2$ . Let  $d$  be the smallest integer  $d > 1$  which is a factor of  $n$ . Show that  $d$  is prime. *Hint:* Towards a contradiction assume  $d$  is not prime, then  $d$  has a factor  $a$ , with  $1 < a < d$  and  $a \mid d$ . Then show  $a \mid n$  and that this gives a contradiction.