Mathematics 300 Homework, December 6, 2017.

In the text on page 214 do problems 1, 3, 5, and 7.

1. The function $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = \dfrac{2-x}{13}$ is bijective. Find its inverse.

2. The function $f\colon (0,1) \to (0,\infty)$ given by $f(x) = \dfrac{1-x}{x}$ is bijective. Find its inverse.

Solution to Problem 1. We wish to solve $f(y) = x$ for y. In this case this is

$$\frac{2-y}{13} = x.$$

This not hard:

$$\begin{aligned}
\frac{2-y}{13} &= x \\
2-y &= 13x && \text{(multiply both sides by 13)} \\
-y &= 13x-2 && \text{(subtract 2 from both sides.)} \\
y &= 2-13x. && \text{(multiply both sides by } -1.)
\end{aligned}$$

Therefore the inverse is

$$f^{-1}(x) = 2-13x.$$

□

Solution to Problem 2. This time we wish to solve

$$f(y) = \frac{1-y}{y} = x$$

for y when $x \in (0,\infty)$. (That is $x > 0$.)

$$\begin{aligned}
\frac{1-y}{y} &= x \\
1-y &= xy && \text{(multiply by } y) \\
-y-xy &= -1 && \text{(rearrange a bit)} \\
y(1+x) &= 1 && \text{(factor out a } y \text{ and multiply by } -1) \\
y &= \frac{1}{1+x} && \text{(divide by } (1+x))
\end{aligned}$$

Therefore the inverse is

$$g^{-1}(x) = \frac{1}{1+x}.$$

□