Solution to Problem 14, Page 100.

We are to show that if $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd and given the hint that we should try cases.

Proof. If n is an integer, then n is either or odd.

Case 1: n is even. Then $n = 2a$ for some $a \in \mathbb{Z}$ and we have

$$\begin{aligned} 5n^2 + 3n + 7 &= 5(2a)^2 + 3(2a) + 7 \\ &= 20a^2 + 6a + 7 \\ &= 2(10a^2 + 3a + 3) + 1 \\ &= 2k + 1 \end{aligned}$$

where $k = 10a^2 + 3a + 1$ is an integer. Therefore $5n^2 + 2n + 7$ is odd.

Case 1: n is odd. Then $n = 2a + 1$ for some $a \in \mathbb{Z}$ and we have

$$\begin{aligned} 5n^2 + 2n + 7 &= 5(2a+1)^2 + 3(2a+1) + 7 \\ &= 5(4a^2 + 4a + 1) + 6a + 3 + 7 \\ &= 20a^2 + 26a + 15 \\ &= 20a^2 + 26a + 15 \\ &= 2(10a^2 + 13a + 7) + 1 \\ &= 2k + 1 \end{aligned}$$

where $k = 10a^2 + 13a + 7$ is an integer. Therefore $5n^2 + 2n + 7$ is odd. □