Mathematics 300 Test 1 Name:

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) For the following list the elements of the set between brackets.

(a) $S = \{x \in \mathbb{Z} : x(x-4) \leq 0\}$ $S =$ $\{0, 1, 2, 3, 4\}$

0 4

(b) $U = \{A : A \subseteq X \text{ and } |A| = 2\}$ where $X = \{1, 2, 4\}$. $U =$ $\{\{1,2\}, \{2,4\}, \{1,4\}\}$

(c) $P = A \times B$ where $A = \{1, 3\}$ and $B = \{x, y\}$. $P =$ $\{(1,x), (1,y), (3,x), (3,y)\}$

2. (10 points) If $A = \{0, 1\}$ and $B = \{3\}$ what are the following

$A \times B = \{(0,3), (1,3)\}$

$\mathcal{P}(A) =$ $\{\emptyset, \{0\}, \{1\}, \{0,1\}\}$

$\mathcal{P}(A \times B) =$ $\{\emptyset, \{(0,3),(1,3)\}, \{(0,3)\}, \{(1,3)\}\}$

$\mathcal{P}(B) = \{\emptyset, \{3\}\}$

$A \times \mathcal{P}(B) =$ $\{(0,\emptyset), (0,\{3\}), (1,\emptyset), (1,\{3\})\}$

3. (15 points) Let $C_j = \{j, j+1, j+2, j+3\}$.

$C_3 = \{3,4,5,6\}$
$C_4 = \{4,5,6,7\}$
$C_5 = \{5,6,7,8\}$

What is $C_3 \cup C_4 \cup C_5$? $\{3,4,5,6,7,8\}$

What is $C_3 \cap C_4 \cap C_5$? $\{5,6\}$

$C_{-2} = \{-2,-1,0,1\}$
$C_{-1} = \{-1,0,1,2\}$
$C_0 = \{0,1,2,3\}$

What is $\bigcup_{k=-2}^{\infty} C_k$? $\{-2,-1,0,1,\ldots\}$

What is $\bigcap_{k=-2}^{\infty} C_k$? $\emptyset$

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15
/99

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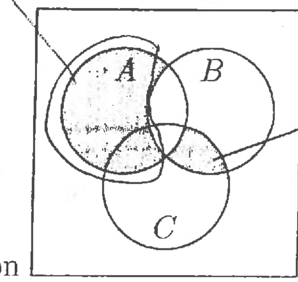
$(A \wedge \overline{B})$



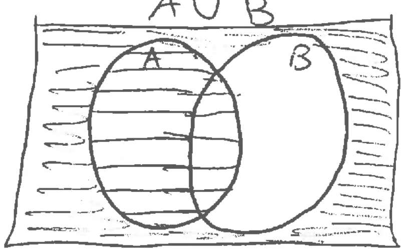
$(B \cap C \wedge \overline{A})$

4. (5 points) Given an expression for the shaded region

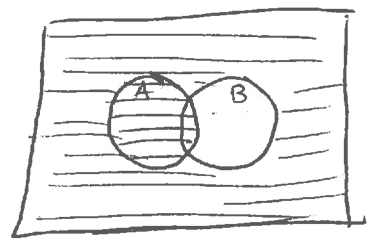
The shaded regions is $(A \wedge \overline{B}) \cup (B \cap C \wedge \overline{A})$

5. (10 points) (a) Draw the Venn diagrams for $A \cup \overline{B}$ and $\overline{B - A}$. (Be sure to label the sets.)

$A \cup \overline{B}$



$B - A$



(b) Is $A \cup \overline{B} = \overline{B - A}$? Why?

yes, as you can see above, the same regions are shaded in both $A \cup \overline{B}$ and $\overline{B - A}$

6. (10 points) (a) Make the truth table for $P \implies \sim Q$.

$P \wedge Q$

P	Q	$\sim Q$	$P \Rightarrow \sim Q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

(b) Make the truth table for $\sim P \vee \sim Q$.

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(c) Are $P \implies \sim Q$ and $\sim P \vee \sim Q$ logically equivalent? Explain your answer.

Yes they are logically equivalent because they have the same truth tables.

7. (5 points) If $S_1 = \{(x, 2-2x) : x \in [0,1]\}$ and $S_2 = \{(x, 2(1-x)) : x \in [1,2]\}$ draw $S_1 \cup S_2$ in the plane.

$2-2x$

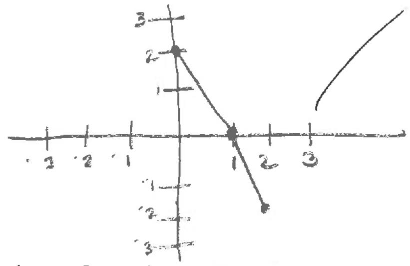
a line $y=mx+b$

$S_1 = \{(0,2)\ (1,0)$

$S_2 = \cancel{1, 2(1-1)}$ $(1,0)$ $2, 2(1-2)$ $2(-1)$ $(2,-2)$ a line $y=mx+b$

2-0

2-2



8. (10 points) Define the following:

(a) The integer n is ***even***. An integer n is even if $n = 2a$ ~~where~~ for some $a \in \mathbb{Z}$.

(b) The integer n is ***odd***. An integer n is odd if $n = 2a+1$ where for some $a \in \mathbb{Z}$.

(c) The integer a ***divides*** the integer b. The integer a divides the integer b, if $b = am$ for some $m \in \mathbb{Z}$ and a is the divisor of b and b is a multiple of a.

(i) How do we write "a divides b" in symbols? a/b

(d) The integer p is ***prime***. An integer p is prime if it has exactly two positive divisors 1 and p. and $n > 1$

9. (10 points) Prove that if x and y are both even integers, then $3x^2 - xy + 5y^2$ is divisible by 4.

Proof

Let x and y be both even integers.

By definition,

$\left.\begin{array}{l} x = 2a \\ y = 2b \end{array}\right\}$ for some $a, b \in \mathbb{Z}$

$3x^2 - xy + 5y^2 = 3(2a)^2 - [(2a)(2b)] + 5(2b)^2$

$3x^2 - xy + 5y^2 = 3(4a^2) - [4ab] + 5(4b^2)$

$3x^2 - xy + 5y^2 = 12a^2 - 4ab + 20b^2$

$3x^2 - xy + 5y^2 = 4(3a^2 - ab + 5b^2)$

$3x^2 - xy + 5y^2 = 4k$ where $k = 3a^2 - ab + 5b^2 \in \mathbb{Z}$.

Thus, $3x^2 - xy + 5y^2$ is divisible by 4

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$a|b \qquad ac|bc$

10. (10 points) Prove that if a, b, and c are integers and a divides b, then ac divides bc.

Lets say $a, b,$ and c are integers.
By definition, if $a|b$, then $b = ad$ for some integer $d \in \mathbb{Z}$, so multiply both sides of $b=ad$ by c

$bc = (ad)(c)$ $\qquad adc$

$= adc$

$= ac(d)$

$= (ac)k$, where $k \in \mathbb{Z}$,

where $k = d \in \mathbb{Z}$,

thus $ac|bc$.

11. (5 points) What is the negotiation of the statement "For every $\varepsilon > 0$ there exists a $N > 0$ such that for all $n \geq N$ the inequality $|a_n| < \varepsilon$ holds."

$\forall_\varepsilon (\varepsilon > 0) \exists_N (N > 0) \forall_n (n \geq N)(|a_n| < \varepsilon)$

$\exists_\varepsilon (\varepsilon > 0) \forall_N (N > 0) \exists_n (n \geq N)(|a_n| \geq \varepsilon)$

There exists an $\varepsilon > 0$, such that for every $N > 0$, there also exists an $n \geq N$ so that $(|a_n| \geq \varepsilon)$.

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