Mathematics 141 Test 1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

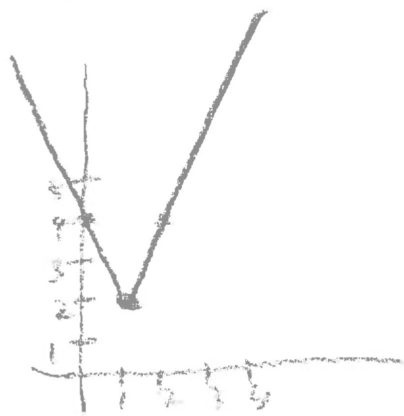
1. (5 points) What are the domain and range of the function $g(x) = \dfrac{1}{x^2}$?

The domain is $\{x : x \neq 0\}$

The range is $\{y : y > 0\}$

2. (5 points) (a) Graph the function

$$f(x) = \begin{cases} 4 - 2x, & x \leq 1; \\ 2x, & x > 1. \end{cases}$$



(b) Define what it means for $f(x)$ to be continuous at the point $x = 1$.

The definition is $\lim_{x \to 1} f(x) = f(1)$

(c) Is this function continuous when $x = 1$? Yes or no. Yes

3. (5 points) Solve the equation $5e^{4t} = 30$. $t = \dfrac{\ln(6)}{4}$

$e^{4t} = \dfrac{30}{5} = 6$

$4t = \ln(6)$

$t = \dfrac{\ln(6)}{4}$

4. (10 points) (a) What is $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta} =$ 1

(b) Compute $\lim_{x \to 0} \dfrac{\tan(3x)}{2x} =$ $\dfrac{3}{2}$

$\lim_{x \to 0} \dfrac{\tan(3x)}{2x} = \lim_{x \to 0} \dfrac{\sin(3x)}{2x \cos(3x)}$

$= \lim_{x \to 0} \dfrac{\sin(3x)}{3x} \dfrac{3}{2\cos(3x)}$

$= 1 \cdot \dfrac{3}{2\cos(0)} = \dfrac{3}{2}$

5. (15 points) Find the following limits.

$\lim_{x \to 3}(4x^2 - x + 5) =$ 38

$= 4(3)^2 - 3 + 5$

$= 36 - 3 + 5 = 38$

$\lim_{t \to 1} \frac{t^2 - 3t + 2}{t - 1} =$ -1

$= \lim_{t \to 1} \frac{(t-1)(t-2)}{(t-1)}$

$= \lim_{t \to 1} (t-2) = 1 - 2 = -1$

$\lim_{x \to 1^-} 4 - \sqrt{1 - x} =$ $4 - \sqrt{0} = 4$

(Note since $x \to 1^-$, that $x < 1$ so $1 - x$ is positive and thus $\sqrt{1-x}$ is defined

$\lim_{x \to \infty} \frac{x^3 - 2x^2 + 1}{4x^3 - 5} =$ $\frac{1}{4}$

$\lim_{x \to \infty} \frac{x^3 - 2x^2 + 1}{4x^3 - 5}$

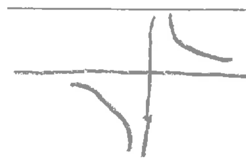
$= \lim_{x \to \infty} \frac{\frac{x^3 - 2x^2 + 1}{x^3}}{\frac{4x^3 - 5}{x^3}}$

$= \lim_{x \to \infty} \frac{1 - \frac{2}{x} + \frac{1}{x^3}}{4 - \frac{5}{x^3}}$

$= \frac{1 - 0 + 0}{4 - 0} = \frac{1}{4}$

$\lim_{x \to 0^-} \frac{4}{x} =$ $-\infty$

Graph



6. (10 points) (a) State the definition of the derivative, $f'(b)$, in terms of a limit.

$f'(b) =$ $\lim_{h \to 0} \frac{f(b+h) - f(b)}{h}$

(b) Let $f(x) = x^2$. Compute $f'(-1)$ using the limit definition and showing all your work.

$f'(-1) = \lim_{h \to 0} \frac{f(-1+h) - f(-1)}{h}$

$= \lim_{h \to 0} \frac{(-1+h)^2 - (-1)^2}{h}$

$= \lim_{h \to 0} \frac{1 - 2h + h^2 - 1}{h}$

$= \lim_{h \to 0} \frac{h(-2+h)}{h}$

$= \lim_{h \to 0} (-2 + h) = -2$

7. (25 points) Compute the derivatives of the following functions:

(a) $f(x) = 5x^3 - 4x^2 + 7x - 19.$ $\qquad f'(x) =$ $15x^2 - 8x + 7$

(b) $w = \frac{3}{x^4} = 3x^{-4}$ $\qquad \frac{dw}{dz} =$ $-12x^{-5} = \frac{-12}{x^5}$

(c) $A(r) = 9\sqrt{r} + 5^2.$ $\qquad A'(r) =$ $\frac{9}{2}r^{-\frac{1}{2}} + 0 = \frac{9}{2\sqrt{r}}$

$= 9r^{\frac{1}{2}} + 25$

(d) $y = 5e^x$ $\qquad \frac{dy}{dx} =$ $5e^x$

(e) $f(t) = 5te^t$ $\qquad f'(t) =$ $5e^t + 5te^t = e^t(5t+5)$

$f'(t) = (5t)'e^t + 5t(e^t)'$

$= 5e^t + 5te^t$

(f) $h(s) = e^s(s^2 + s + 1)$ $\qquad h'(s) =$ $e^s(s^2+3s+1)$

$h'(s) = (e^s)'(s^2+s+1) + e^s(s^2+s+1)'$

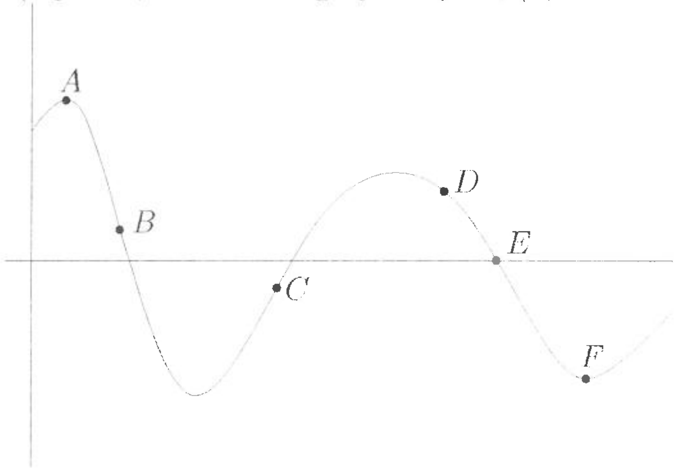
$= e^s(s^2+s+1) + e^s(2s+1) = e^s(s^2+3s+1)$

(g) $y = \frac{e^x + 2}{x + 1}$ $\qquad y' =$ $\frac{xe^x - 2}{(x+1)^2}$

$y' = \frac{(e^x+2)'(x+1) - (e^x+2)(x+1)'}{(x+1)^2}$

$= \frac{e^x(x+1) - (e^x+2)(1)}{(x+1)^2} = \frac{xe^x + \cancel{e^x} - \cancel{e^x} - 2}{(x+1)^2}$

8. (5 points) This is the graph of $y = f(x)$.



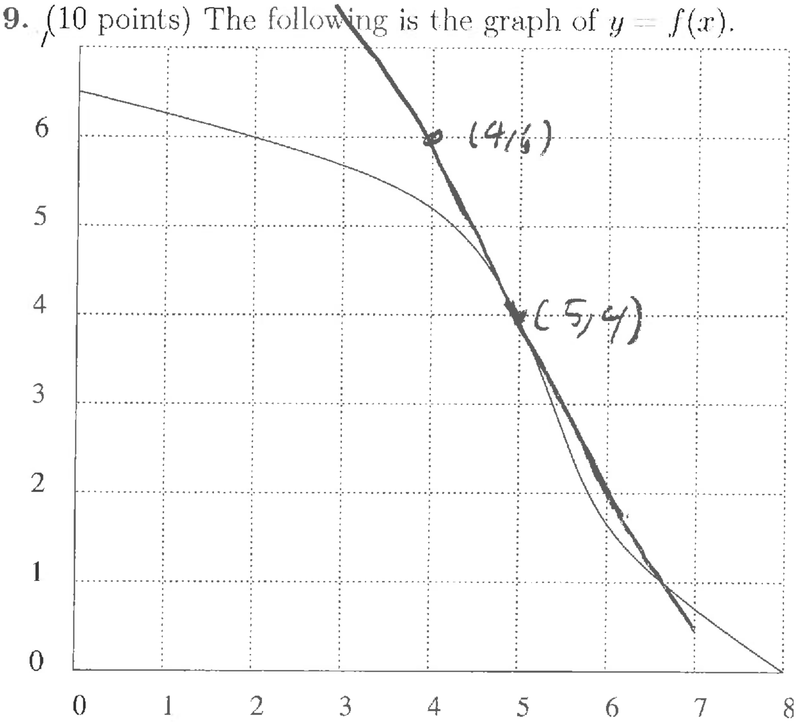
At which point(s) is $f(x) < 0$? C, F

At which point(s) is $f(x) = 0$? E

At which point(s) is $f'(x) < 0$? B, E

At which point(s) is $f'(x) = 0$? A, F

9. (10 points) The following is the graph of $y = f(x)$.



slope of tangent line

$= \frac{\text{rise}}{\text{run}} = \frac{6-4}{4-5} = \frac{2}{-1} = -2$

$\approx f'(5)$

(a) What is the value of $f(5)$? $f(5) =$ $\underline{\quad 4 \quad}$

(b) Draw the tangent line to $y = f(x)$ at the point where $x = 5$, label two points on it and use these points to estimate $f'(5)$.

$f'(5) \approx$ $\underline{\quad -2 \quad}$

10. (10 points) For the function $y = 1 + x - x^2$

(a) Find the value of the derivative of the function when $x = 2$.

$y' = \frac{dy}{dx} = 1 - 2x$

$y'(2) = \left.\frac{dy}{dx}\right|_{x=2} = 1 - 2(2) = -3$

$\left.\frac{dy}{dx}\right|_{x=2} =$ $\underline{\quad -3 \quad}$

(b) What is the equation of the tangent line to the graph at the point where $x = 2$?

$y - y_0 = m(x - x_0)$

$x_0 = 2$

$m = -3$

$y_0 = 1 + 2 - (2)^2$

$= 3 - 4 = -1$

$y - (-1) = -3(x-2)$

The equation is $\underline{\quad y + 1 = -3(x-2) \quad}$

or $y = -3x + 5$