Mathematics 141 Test 3 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (25 points) Compute the following indefinite integrals.

(a) $\int (4x^5 + 3x^4 - 9x^2 + 7)\, dx$ The integral is $\frac{2}{3}x^6 + \frac{3}{5}x^5 - 3x^3 + 7x + C$

$= \frac{4}{6}x^6 + \frac{3}{5}x^5 - \frac{9}{3}x^3 + 7x + C$

(b) $\int \big(6\sin(2\theta) + 12\cos(3\theta)\big)\, d\theta$ The integral is $-3\cos(2\theta) + 4\sin(3\theta) + C$

$= -6\frac{\cos(2\theta)}{2} + 12\frac{\sin(3\theta)}{3} + C$

(c) $\int \left(3\sqrt{t} + \frac{4}{t^3} - \frac{4}{t}\right) dt$ The integral is $2t^{3/2} - 2t^{-2} - 4\ln|t| + C$

$= \int (3t^{\frac{1}{2}} + 4t^{-3} - \frac{4}{t})\,dt$

$= 3\left(\frac{2}{3}\right)t^{\frac{3}{2}} - \frac{4t^{-2}}{+2} - 4\ln|t| + C$

(d) $\int 4e^{3x}\, dx$ The integral is $\frac{4}{3}e^{3x} + C$

(e) $\int \tan x \, dx$ The integral is $\ln|\sec x| + C$

(f) $\int \frac{3}{\sqrt{2t+1}} \, dt$ $u = 2t+1$ The integral is $3u^{\frac{1}{2}} + C$

$du = 2\,dt$

$\frac{1}{2} du = dt$

$= \int \frac{\frac{3}{2} du}{\sqrt{u}}$

$= \frac{3}{2} \int u^{-\frac{1}{2}} du = \frac{3}{2}(2) u^{\frac{1}{2}} + C$

(g) $\int \frac{dz}{1+z^2}$ The integral is $\tan^{-1}(z) + C$

(h) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} \, dt$ The integral is $\frac{1}{2\cos(2t+1)} + C$

$u = \cos(2t+1)$

$du = -\sin(2t+1)(2)\,dt$

$-\frac{1}{2} du = \sin(2t+1)\,dt$

$= \int \frac{-\frac{1}{2} du}{u^2}$

$= -\frac{1}{2} \int u^{-2} du$

$= -\frac{1}{2}(-1) u^{-1} + C$

$= \frac{1}{2u} + C = \frac{1}{2\cos(2t+1)} + C$

2. (20 points) Compute the following definite integrals.

(a) $\int_0^2 (6x^2 - 4x + 3)\, dx$ The integral is 14

$= (2x^3 - 2x^2 + 3x)\Big|_0^2$

$= 2(2)^3 - 2(2)^2 + 3(2) - 0$

$= 16 - 8 + 6$

$= 14$

(b) $\int_0^{\pi/2} \sin(2\theta)\, d\theta$ The integral is 1

$= -\frac{\cos(2\theta)}{2}\Big|_0^{\pi/2}$

$= -\frac{\cos(\pi)}{2} + \frac{\cos(0)}{2}$

$= \frac{-(-1)}{2} + \frac{1}{2} = 1$

(c) $\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx$ The integral is 4

$u = x^2 + 1$

$du = 2x\,dx$

$2du = 4x\,dx$

$x = 0 \Rightarrow u = 0^2 + 1 = 1$

$x = \sqrt{3} \Rightarrow u = (\sqrt{3})^2 + 1 = 4$

$= \int_1^4 \frac{2\,du}{\sqrt{u}}$

$= 2\int_1^4 u^{-\frac{1}{2}}\,du = 2(2)u^{\frac{1}{2}}\Big|_0^4 = 4(4^{\frac{1}{2}}) - 4(1^{\frac{1}{2}})$

$= 4 \cdot 2 - 4 = 4$

(d) $\int_{\sqrt{2}}^1 \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du$ The integral is $-3/4$

$= \int_{\sqrt{2}}^1 (\frac{1}{2}u^7 - 5u^{-5})\,du$

$= (\frac{1}{2}\frac{u^8}{8} + \frac{5}{4}u^{-4})\Big|_{\sqrt{2}}^1$

$= (\frac{1}{16} + \frac{5}{4}) - (\frac{1}{2}\frac{(\sqrt{2})^8}{8} + \frac{5}{4}(\sqrt{2})^{-4})$

$= \frac{1}{16} + \frac{20}{16} - (\frac{16}{16} + \frac{20}{64})$

$= -\frac{3}{4}$

(e) $\int_1^e \frac{\ln(z)}{z}\, dz$ The integral is $\frac{1}{2}$

$u = \ln(z)$

$du = \frac{dz}{z}$

$z = 1 \Rightarrow u = \ln(1) = 0$

$z = e \Rightarrow u = \ln(e) = 1$

$= \int_0^1 u\,du = \frac{1}{2}$

3. (10 points) For the graph at right which of the labeled points have

$y < 0$ ______ A, E

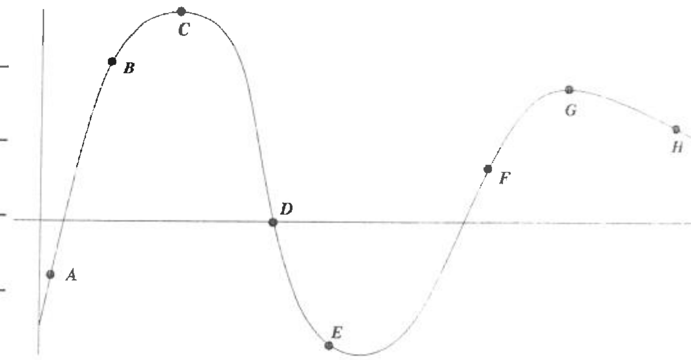
$y = 0$ ______ D

$y' > 0$ ______ A, B, F

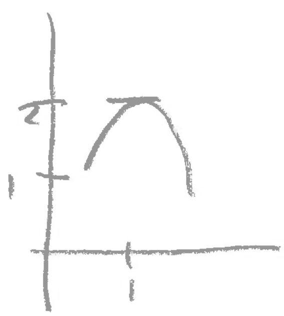
$y'' > 0$ ______ E

$y < 0$ and $y'' < 0$ ______ A

A local maximum ______ C, G.



4. (10 points) (a) Draw the graph of a function $y = f(x)$ with $f(1) = 2$, $f'(1) = 0$, and $f''(1) < 0$.



(b) For the graph you have drawn (circle one):

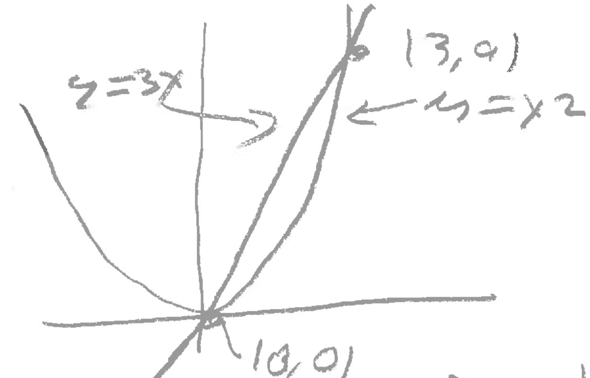
$x = 1$ is a local maximizer. $x = 1$ is a local minimizer. Not enough information to say.

5. (10 points) Compute the following:

(a) $\dfrac{d}{dx} \displaystyle\int_0^x \sin(t^2)\,dt =$ ______ $\sin(x^2)$

(b) $\dfrac{d}{dx} \displaystyle\int_{-3}^{e^x} \sin(t^2)\,dt = \sin((e^x)^2)(e^x)'$ ______ $e^x \sin(e^{2x})$

6. (10 points) Find the area between the graphs of $y = 3x$ and $y = x^2$.



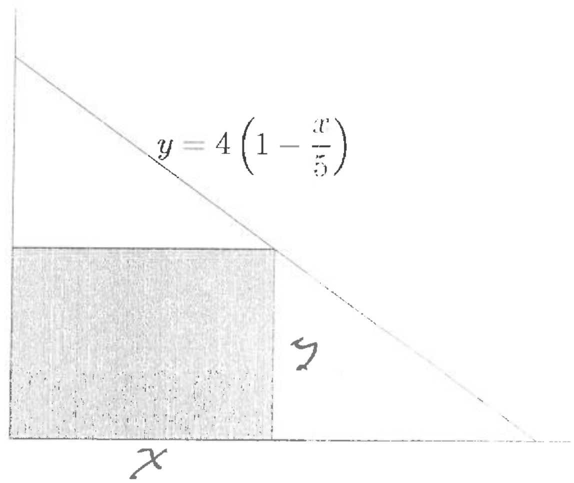
The area is $\frac{9}{2}$

$$\text{Area} = \int_0^3 (3x - x^2)\,dx$$
$$= \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)\Big|_0^3$$
$$= \frac{3\cdot(3)^2}{2} - \frac{3^3}{3} = \frac{27}{2} - 9$$
$$= \frac{27-18}{2}$$
$$= \frac{9}{2}$$

To find points of intersection set $x^2 = 3x$

$$x^2 = 3x$$
$$x^2 - 3x = 0$$
$$x(x-3) = 0$$
$$x = 0, 3$$
$$x = 0 \Rightarrow y = 0$$
$$x = 3 \Rightarrow y = 9$$

7. (10 points) One corner of a rectangle is on the line $y = 4\left(1 - \frac{x}{5}\right)$ as shown in the figure. What are dimensions of the rectangle that maximizes the are of the rectangle.



Length of base: $x = 2.5$

Height: $y = 2$

Maximum area: $A = 5$

The area is

$$A = xy = x\,4\left(1 - \frac{x}{5}\right) = 4x\left(1 - \frac{x}{5}\right)$$
$$A = 4x - \frac{4x^2}{5}$$
$$\frac{dA}{dx} = 4 - \frac{8x}{5} = 0$$
$$-\frac{8x}{5} = -4$$
$$x = \frac{4\cdot 5}{8} = \frac{5}{2} = 2.5$$
$$y = 4\left(1 - \frac{x}{5}\right) = 4\left(1 - \frac{2.5}{5}\right) = 2$$
$$A = xy = \left(\frac{5}{2}\right)(2) = 5$$

8. (10 points) For the function $y = x^3 - 12x + 3$. (In this problem be sure to show all your work.)
(a) Find the local maximums and minimums

Local maximums (give (x, y) coordinates): $(-2, 19)$

Local minimums (give (x, y) coordinates): $(2, -13)$

$y' = 3x^2 - 12$
$= 3(x^2 - 4)$
$= 3(x-2)(x+2) = 0$

So $x = 2, -2$ are the critical points

$y' = 3(x-2)(x+2)$: $+++$ on $(-\infty, -2)$, $----$ on $(-2, 2)$, $++++++$ on $(2, \infty)$

max. at -2, min at 2

loc. max $x = -2$
$y = -8 + 24 + 3 = 19$

loc. min $x = 2$,
$y = 8 - 24 + 3 = -13$

(b) What is the inflection point(s)?

Inflection point(s) are (give (x, y) coordinates): $(0, 3)$

$y'' = 3x = 0$
so inflection point is where $x = 0$

y'': $--$ before 0, $+++$ after 0

(c) Sketch a graph of the function.