Mathematics 122

Quiz 13 Name: Key

You must show your work to get full credit.

1. Let $P(t)$ have exponential growth with $P(0) = 40$ and $P(5) = 60$. Find a formula for $P(t)$.

Method 1 $P(t) = P_0 a^t = 40a^t$

$P(5) = 40a^5 = 60$ $a = \left(\frac{60}{40}\right)^{\frac{1}{5}} = 1.0845$

$P(t) =$ $40(1.0845)^t = 40e^{.08109t}$

2. $500 dollars is invested at 13% simple interest.

(a) What is the principle after t years? $P(t) =$ $500(1.13)^t$

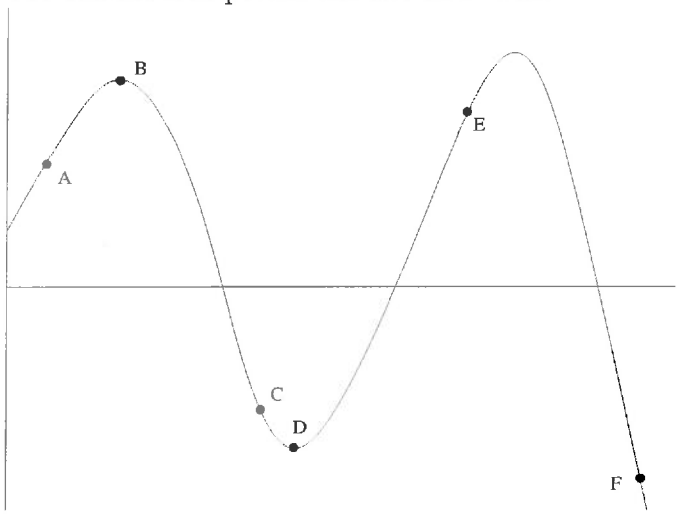
(b) How long until it reaches $5,000? Time to \$5,000. 18.840 years

solve $500(1.13)^t = 5000$

$(1.13)^t = 10$

$t\ln(1.13) = \ln(10)$ $\rightarrow t = \frac{\ln(10)}{\ln(1.13)} = 18.840$

3. For the labeled points list the ones with



$f > 0$ A, B, E

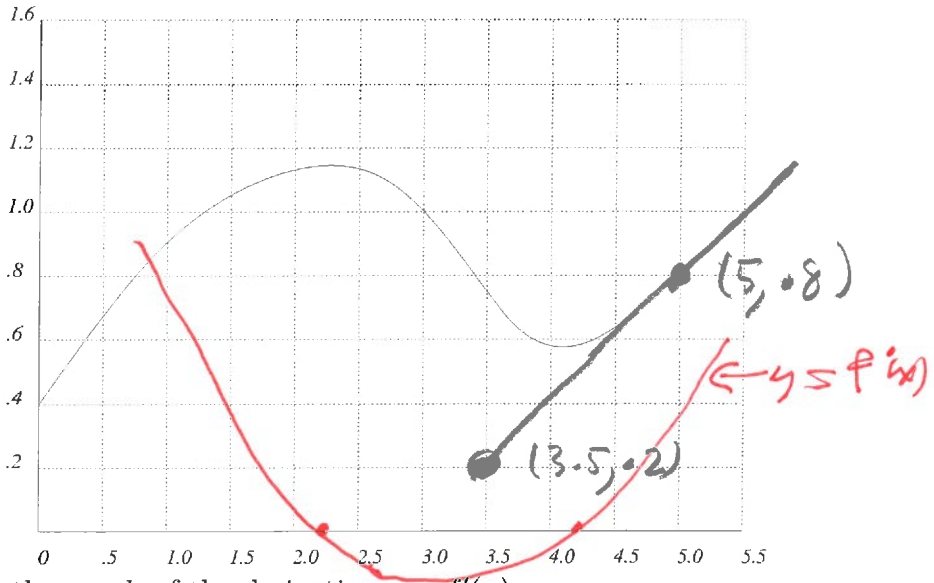
$f' < 0$ C, F

$f' = 0$ B, D

$f'' > 0$ C, D

Concave down. A, B, E, D

4. In the following graph of $y = f(x)$ draw the tangent line at the point where $x = 5.0$, label two points on this line and use them to estimate $f'(5)$.



$f'(5) \approx$ .4

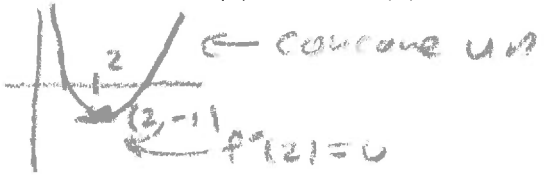
slope $= \frac{.8 - .2}{5 - 3.5}$

$= \frac{.6}{1.5} = .4$

5. On the graph of the last problem draw the graph of the derivative $y = f'(x)$.

6. (a) Draw a graph of a function $y = f(x)$ that satisfies $f'(x) < 0$ for $-1 < x < 2$, $f'(x) > 0$ for $x < -1$ or $x > 2$, $f'(-1) = f'(2) = 0$.

(b) Draw a graph of a function with $f(2) = -1$, $f'(2) = 0$, and $f''(x) > 0$.



← concave up

7. If $f(3) = 9$ and $f'(3) = -2$ estimate the following

$f(3.1) \approx$ 8.8

$= f(3) + f'(3)(.1)$
$= 9 + (-2)(.1)$
$= 8.8$

$f(2.98) \approx$ 9.04

$= f(3 + (-.02))$
$\approx f(3) + f'(3)(-.02)$
$= 9 + (-2)(-.02) = 9.04$

8. Compute the following derivatives, where a and b are constants.

(a) $f(x) = 4x^5 - 9x^2 + 7x - 2$. $f'(x) =$ $20x^5 - 18x + 7$

(b) $A = 5\sqrt{r} - \frac{4}{r^7} = 5r^{\frac{1}{2}} - 4r^{-7}$ $\frac{dA}{dr} =$ $\frac{5}{2}r^{-\frac{1}{2}} + 28r^{-8}$

(c) $f(t) = 4a^3 + \frac{5}{t^2}$. $= 4a^3 + 5t^{-2}$ $f'(t) =$ $-10t^{-3}$

$(4a^3)' = 0$ as a is constant

(d) $w = \frac{z + 9z^4}{z^2}$. $= \frac{z}{z^2} + \frac{9z^4}{z^2}$ $\frac{dw}{dz} =$ $-z^{-2} + 18z$

$= z^{-1} + 9z^2$

9. What is the second derivative of $s(t) = 100 + 5t - 16t^2$?

$s'(t) = 5 - 32t$

$s''(t) =$ -32

10. What is the tangent line to $y = x^2 - 2$ at the point where $x = 1$?

$y(1) = 1^2 - 2 = -1 = y_0$

$y' = 2x$

$y'(1) = 2 \cdot (1) = 2 = m$

Point slope form

$y - y_0 = m(x - x_0)$

$y - (-1) = 2(x - 1)$

$y + 1 = 2x - 2$

$y = 2x - 3$

Tangent line is $y = 2x - 3$