Mathematics 122

Quiz 26

Name: Key

You must show your work to get full credit.

1. Let a be a positive constant. Find the maximum and maximizer of $f(x) = x^3(a - x)$ on the interval $0 \le x \le a$.

Maximizer is $\frac{3a}{4}$

Maximum is $\frac{3^3a^4}{4^4} = \frac{27a^4}{256} = (.037)a^4$

$f(0) = f(a) = 0$ and $f(x) > 0$ for $0 < x < a$ so graph will look like

$f(x) = ax^3 - x^4$

$f'(x) = 3ax^2 - 4x^3$
$= x^2(3a - 4x) = 0$

critical points are $x = 0$, $x = \frac{3a}{4}$.

Thus $x = \frac{3a}{4}$ is maximizer

$f(\frac{3a}{4}) = (\frac{3a}{4})^3(a - \frac{3a}{4}) = \frac{3^3a^3}{4^3}(\frac{a}{4}) = \frac{3^3a^3}{4^4}$

2. (a) Define what it means for $x = a$ to be a critical point of $y = f(x)$.

$f'(a) = 0$ or $f'(a)$ is undefined

(b) Find the derivative of $f(x) = x^2e^{-x}$

$f'(x) = (x^2)'e^{-x} + x^2(e^{-x})'$
$= 2xe^{-x} - x^2e^{-x} = (2x - x^2)e^{-x}$

$f'(x) =$ $(2x - x^2)e^{-x}$

(c) Find the critical points of $f(x)$.

The critical points are: $0, 2$

solve
$(2x - x^2)e^{-x} = 0$
factor
$x(2 - x)e^{-x} = 0$

so $x = 0$, $x = 2$

(d) Find $f''(x)$. $f''(x) = (2x - x^2)'e^{-x} + (2x - x^2)(e^{-x})'$

$= (2 - 2x)e^{-x} - (2x - x^2)e^{-x}$
$= (2 - 2x - 2x + x^2)e^{-x}$

$f''(x) =$ $(2 - 4x + x^2)e^{-x}$

(e) Use $f''(x)$ to determine which of the critical points are local maximizers and which are local minimizers.

Maximizers: 2

Minimizers: 0

$f''(0) = (2 - 4(0) + 0^2)e^{-0} = 2 > 0$ so $x = 0$ is minimizer
$f''(2) = (2 - 4(2) + 2^2)e^{-2} = -2e^{-2} < 0$, so $x = 2$ is maximizer