Mathematics 122 Test #1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (10 points) A government bond cost \$500 and pays 15% simple interest.

(a) What is it worth in 10 years? It is worth \$ 2022.78

$500(1.15)^{10} = 2022.78$

(b) How long does it take the bond to double in value?

In t years it was the value $P(t) = 500(1.15)^t$

so solve $500(1.15)^t = 2 \cdot 500$

$(1.15)^t = 2$

$t \ln(1.15) = \ln(2)$

$t = \frac{\ln(2)}{\ln(1.15)} = 4.959$

Time to double is 4.959 years

2. (10 points) The variables p and q are related as in the table

p	10.0	10.5	11.0	11.5
q	14.5	13.0	11.5	10.0

(a) Explain why the relation between p and q could be linear. (This will involve both doing some calculations and writing at least one sentence explaining why the calculations are relevant.)

slopes at

① $\frac{\Delta q}{\Delta p} = \frac{13 - 14.5}{10.5 - 10} = \frac{-1.5}{.5} = -3$

② $\frac{\Delta q}{\Delta p} = \frac{11.5 - 13}{11 - 10.5} = \frac{-1.5}{.5} = -3$

③ $\frac{\Delta q}{\Delta p} = \frac{10 - 11.5}{11.5 - 10} = \frac{-1.5}{.5} = -3$

Thus the slopes are all the same. So it is linear.

(b) Find q as a function of p.

$\frac{\Delta q}{\Delta p} = \frac{q - 14.5}{p - 10} = -3$

$q - 14.5 = -3(p - 10)$

$q = -3p + 30 + 14.5$

$q = -3p + 44.5$

(c) What is the value of q when $p = 10.7$? 12.4

$q = -3(10.7) + 44.5 = 12.4$

3. (5 points) If $f(5) = 15$ and $f'(5) = 1.5$ estimate the following

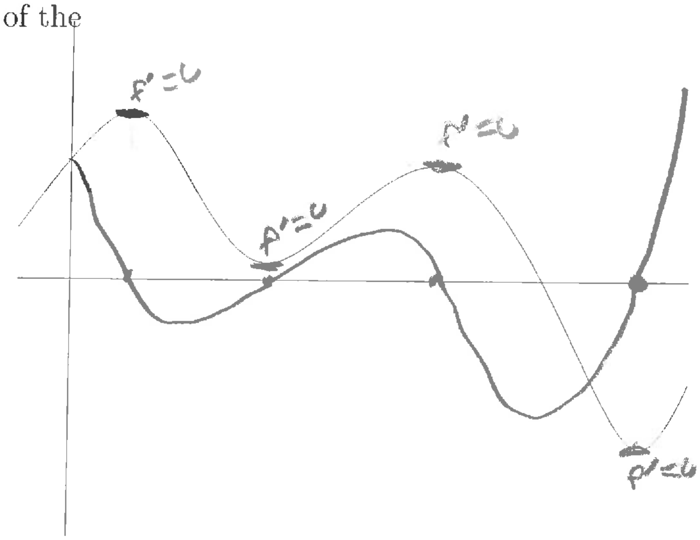
$f(5.15) \approx$ 15.225

$f(5.15) \approx f(5) + f'(5)(.15)$
$= 15 + 1.5(.15)$
$= 15.225$

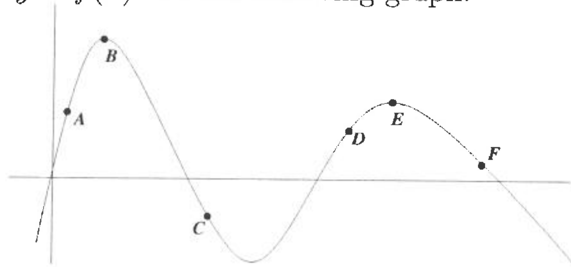
$f(4.99) \approx$ 14.985

$f(4.99) \approx f(5) + f'(5)(-.01)$
$= 15 - 1.5(.01)$
$= 14.985$

4. (5 points) On the same axis draw the graph of the derivative $y = f'(x)$.



5. (10 points) The function $y = f(x)$ has the following graph.



At which of the labeled points is $f > 0$? A, B, D, E, F

At which of the labeled points is $f' > 0$? A, D

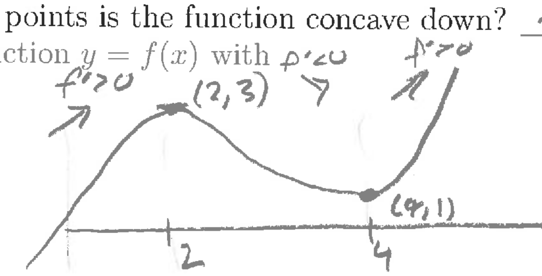
At which of the labeled points is $f' = 0$? B, E

At which of the labeled points is $f'' > 0$ C

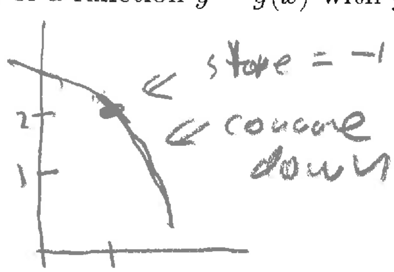
At which of the labeled points is the function concave down? A, B, D, E, F

6. (10 points) (a) Draw the graph of a function $y = f(x)$ with

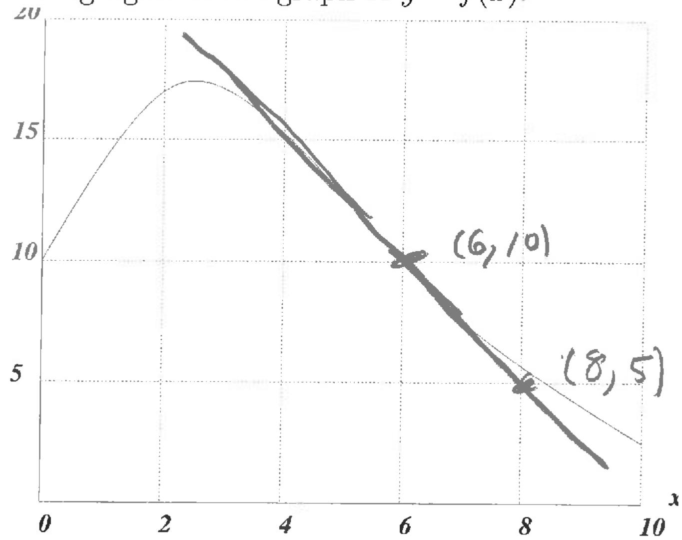
- $f(2) = 3$ and $f(4) = 1$,
- $f'(x) < 0$ for $2 < x < 4$,
- $f'(x) > 0$ for $x < 2$ and $x > 4$, and
- $f'(2) = f'(4) = 0$.



(b) Draw the graph of a function $y = g(x)$ with $g(1) = 2$, $g'(1) = -1$, and $g''(x) < 0$.



7. (10 points) In the following figure is the graph of $y = f(x)$.



What is the value of $f(6)$? $\underline{\quad 10 \quad}$

What is the average rate of change between $x = 2$ and $x = 8$? $\underline{\quad -1 \quad}$

$f(2) \approx 12 \quad \frac{\Delta y}{\Delta x} = \frac{6-12}{8-2} = \frac{-6}{6} = -1$

$f(8) \approx 6$

Draw the tangent line to the graph at the point where $x = 6$, label two points on that line and use those points to estimate the derivative $f'(6)$.

$f'(6) =$ slope of tangent line

$= \frac{5-10}{8-6} = \frac{-5}{2} = -2.5$

$f'(6) \approx \underline{\quad -2.5 \quad}$

8. (10 points) The cost, C, in dollars of producing apple cider is a function of the number, ℓ, of liters produced. That is $C = f(\ell)$. If

$$f(100) = 65, \qquad f'(100) = .32$$

(a) In $f(100) = 65$

What are the units of 100? $\underline{\quad \text{liters} \quad}$

What are the units of 65? $\underline{\quad \text{dollars} \quad}$

(b) In $f'(100) = .32$

What are the units of 100? $\underline{\quad \text{liters} \quad}$

What are the units of .32? $\underline{\quad \text{dollars/liter} \quad}$

(c) Use these numbers to estimate $f(102)$.

$f(102) \approx \underline{\quad 65.64 \quad}$

$f(102) \approx f(100) + f'(100)(2)$

$= 65 + .32(2)$

$= 65.64$

9. (10 points) The following table gives some values for $y = f(x)$.

x	1.0	1.2	1.4	1.6
$f(x)$	5.2	4.9	4.6	4.0

(a) What is the average rate of change between $x = 1.2$ and $x = 1.6$?

Average rate of change is -2.25

$$\frac{\Delta y}{\Delta x} = \frac{4.0 - 4.9}{1.6 - 1.2} = \frac{-.9}{.4} = -2.25$$

(b) Estimate $f'(1.3)$.

$f'(1.3) \approx -1.5$

$$f'(1.3) \approx \frac{\Delta y}{\Delta x} = \frac{4.6 - 4.9}{1.4 - 1.2}$$
$$= \frac{-.3}{.2} = -1.5$$

10. (5 points) Find the tangent line to $y = x^2 + 2$ at the point where $x = 1$.

The equation of the tangent line is $y = 2x + 1$

Point slope form is $y - y_0 = m(x - x_0)$

$x_0 = 2$

$y_0 = y(1) = 1^2 + 2 = 3$

$y' = 2x$

$m = y'(1) = 2 \cdot 1 = 2$

$y - 3 = 2(x - 1)$

$y = 2x - 2 + 3$

$= 2x + 1$

11. (10 points) Let a and b be constants. Compute the derivatives of the following functions.

(a) $f(x) = 9x^4 - 5x^2 + 2x - 17$.

$f'(x) = 36x^3 - 10x^2 + 2$

(b) $A = \frac{2}{r^3} - 2\sqrt{r}$. $= 2r^{-3} - 2r^{\frac{1}{2}}$

$\frac{dA}{dr} = -6r^{-4} + 1r^{-\frac{1}{2}}$

$\frac{dA}{dr} = -6r^{-4} + r^{-\frac{1}{2}}$

(c) $V = \frac{3}{a^4} + \pi b r^2$.

$V' = \left(\frac{3}{a^4}\right)' + (\pi b r^2)' = 0 + 2\pi b r$

$\frac{dV}{dr} = 2\pi b r$

12. (5 points) What is the second derivative of $f(x) = x^3 + \frac{4}{x}$?

$f(x) = x^3 + 4x^{-1}$

$f'(x) = 3x^2 - 4x^{-2}$

$f''(x) = 6x + 8x^{-3}$

$f''(x) = 6x + 8x^{-3}$