Mathematics 122 Test #2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (20 points) Compute the derivatives of the following functions.

(a) $h(t) = c_2t^2 + c_1t + c_0$ where c_0, c_1, and c_2 are constants.

$h'(t) =$ $2c_2t + c_1$

(b) $A(r) = 5e^{2r}$

$A'(r) = 5e^{2r}(2)$
$= 10e^{2r}$

$A'(r) =$ $10e^{2r}$

(c) $w = 2z^3 \ln(z)$

$w' = (2z^3)' \ln(z) + 2z^3(\ln(z))'$
$= 6z^2 \ln(z) + 2z^3 \frac{1}{z}$

$\frac{dw}{dz} =$ $6z^2 \ln(z) + 2z^2$

(d) $f(x) = 3\sqrt{x^2 + x} = 3(x^2+x)^{\frac{1}{2}}$

$f'(x) = \frac{3}{2}(x^2+x)^{-\frac{1}{2}}(2x+1)$

$f'(x) =$ $\frac{3}{2}(x^2+x)^{-\frac{1}{2}}(2x+1)$
$= \frac{2(x+1)}{2\sqrt{x^2+1}}$

(e) $y = \dfrac{ax}{x+b}$ with a and b constants.

$\frac{dy}{dx} =$ $\frac{ab}{(x+b)^2}$

$y' = \frac{(ax)'(x+b) - ax(x+b)'}{(x+b)^2} = \frac{a(x+b) - ax(1)}{(x+b)^2} = \frac{ax+ab-ax}{(x+b)^2}$

2. (10 points) A student organization makes coffee mugs with the USC logo on them to sell. Assume the cost of making 100 mugs is $C(100) = \$400.00$ and the marginal cost for the 100^{th} mug is $MC(100) = \$2.50$.

(a) Estimate the total cost of making 103 mugs.

$C(103) \approx$ \$407.5

$C(103) \approx C(100) + C'(100)(3)$
$= C(100) + MC(100)(3)$
$= 400 + 3(2.50)$
$= 407.5$

(b) If $MR(100) = \$2.25$ should the organization **increase** or **decrease** (circle one). Write a sentence or two explaining why.

The marginal profit is $M\pi(100) = MR(100) - MC(100)$
$= 2.25 - 2.50$
$= -.25$

so they lose 25¢ by selling another coffee mug.

3. (15 points) For the function

$$f(x) = \frac{x^3 - 11x^2 + 34x - 14}{3 + x^2}$$

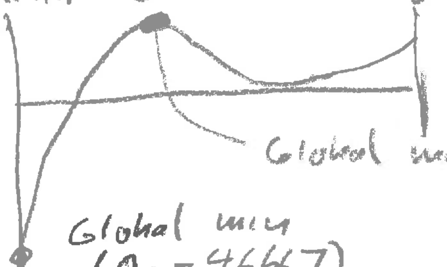
on the interval $0 \leq x \leq 8$ find the following and explain how you did it on the calculator.

Global maximizer 1.4373   Global maximum 2.9837

Global minimizer 0   Global minimum −4.66667

\ Y1 = (X^3−11X^2+34X−14)/(3+X^2)

Xmin = 0   Xmax = 8   ZoomFit



Global max 2nd calc 4: maximum

X = 1.43734

Y = 2.98337

Global min

(0, −46667)

4. (10 points) Find the global maximum and maximizer of $f(x) = x^2(c-x)$ on the interval $0 \leq x \leq c$ where c is a positive constant.

Maximizer $\frac{2}{3}c$ Maximum $\frac{4c^3}{27}$

$f(x) = x^2(c-x)$. $f(0) = f(c) = 0$.

so max is not at endpoint.

$f(x) = cx^2 - x^3$

$f'(x) = 2cx - 3x^2$

$= x(2c - 3x) = 0$

so critical points are

$x = 0$ and where $2x - 3x = 0$ →

$-3x = -2c$

$x = \frac{2}{3}c$

Then

$f(\frac{2}{3}c) = (\frac{2c}{3})^2(c - \frac{2}{3}c)$

$= \frac{4c^2}{9}(\frac{c}{3})$

$= \frac{4c^3}{27}$

5. (5 points) Find the inflection point(s) of $f(x) = x^3 - 12x^2 - 5x + 1$.

Inflection points are: $x = 4$.

Find where 2nd derivative changes sign.

$f'(x) = 3x^2 - 24x - 5$

$f''(x) = 6x - 24 = 0$

$6x = 24$

$x = \frac{24}{6}$

6. (5 points) If $V(13) = 45$ and $V'(13) = -2$ estimate $V(13.2)$.   44.6 = $V(13.2) \approx$

44.6

$V(13.2) \approx V(13) + V'(13)(.2)$

$= 45 + (-2)(.2)$

$= 44.6$

7. (10 points) (a) Define what it means for a point, $x = a$, to be a critical point of a function $f(x)$.

$f'(a) = 0$ or $f'(a)$ is undefined.

(b) Let $f(x)$ be the function $f(x) = x^3 - 3x$. find the first derivative and use it to find the critical points of $f(x)$.

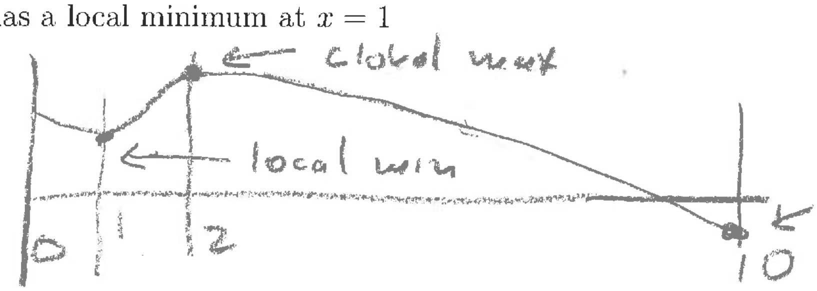
$f'(x) = 3x^2 - 3$
$= 3(x^2 - 1)$
$= 3(x-1)(x+1) = 0$
solving gives $x = 1, -1$

$f'(x) =$ $3x^2 - 3$

The critical points are: $1, -1$

8. (10 points) Graph a function on the interval $0 \le x \le 10$ such that

- $f(x)$ has a global maximum at $x = 2$,
- $f(x)$ has a global minimum at $x = 10$,
- $f(x)$ has a local minimum at $x = 1$



9. (15 points) The revenue brought in by sell a quantity q of an item is $R(q) = 50q$ and the cost of producing q items is $C(q) = 100 + .1q^2$.

(a) What is the profit of producing q items? $\pi(q) =$ $50q - (100 + .1q)$

$\pi(q) = R(q) - C(q)$
$= 50q - (100 + .1q)$

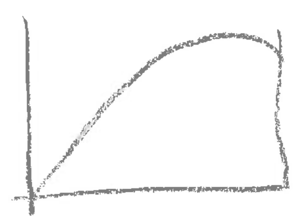
(b) How many items should be produced to maximize profit? Be sure to explain how you got your answer.

Graph the profit function
$Y1 = 50 - (100 + .1X^2)$

Xmin = 0
Xmax = 300 (Used some trial and error to find this)

ZoomFit

$q =$ 250



2nd calc maximum
$X = 249.9999$
$= 250$
$Y = 6150$

(c) What is the maximum profit? Maximum $\pi(q)$ is 6150