Mathematics 122 Test #3 Name: Key

1. (10 points) Let a be a positive constant and let $f(x)$ be the function

$$f(x) = x(a - x) \quad = ax - x^2$$

(a) Compute the first and second derivatives of $f(x)$.

$f'(x) =$ $a - 2x$ $\qquad\qquad$ $f''(x) =$ -2

(b) What is the critical point of $f(x)$? $\qquad$ The critical point is $a/2$

Solve $f'(x) = a - 2x = 0$

$2x = a$

$x = a/2$

(c) Is the critical point a maximizer or minimizer? Explain how you determined this.

Maximizer or minimizer: Maximizer.

Why? (*Hint:* Thee are many ways to do this, one is to look at the second derivative and see what it says about the function being concave up or down.)

$f''(x) = -2 < 0$ so concave down.

Thus a maximum

2. (10 points) The driver of a car going 44 feet per second (which is 30 miles per hour) sees a squirrel in the road and the driver hits the brakes. The following table gives the velocity, v, of the car in ft/sec as a function of t, the time in seconds since the brakes were applied.

t	0	.5	1.0	1.5	2.0
v	44	33	22	11	0

$\Delta t = .5$

(a) Give upper and lower bounds for the distance traveled by the car since the brakes were applied. Show your work.

Lower bound $\qquad$ Upper bound

$.5(33+22+11+0) = 33$ $\qquad$ $.5(44+33+22+11) = 55$

(b) When the brakes are applied the squirrel is 40 feet from the car. If the squirrel is blinded by the car's head lights and does not move what is your best guess at what happens to it and explain why you think this. Best guess at distance covered is the average of upper and lower bounds $= \frac{55+33}{2} = 44$ ft.

So the best guess is that the squirrel gets hit.

3. (15 points) A small the owner of a furniture shop starts to produce small book cases to sell to college students. The fixed costs to start the production of these bookcases is \$1,500. The marginal cost function for producing them is

$$MC(q) = C'(q) = \frac{200}{4 + .3q}$$

and she sells them for \$95 each.

(a) What is the cost of producing 40 of the bookcases? $C(40) =$ 2424.20

$C(0) =$ fixed costs $= \$1500$

$C(40) = C(0) + \int_0^{40} C'(q)\,dq = 1500 + \int_0^{40} \frac{200}{4+.3q}\,dq = 2424.20$

(b) What is the revenue function? $R(q) =$ \$95q

$R(q) = q \times$ (cost of each book case)

$= 95q$

(c) What is the profit of producing 40 bookcases? $\pi(40) =$ \$1375.80

$\pi(40) = R(40) - C(40)$

$= 95 \times 40 - 2424.20 = 1375.80$

4. (10 points) Use your calculator to calculator to compute the following, and also write down what you punched into your calculator to get the answer.

fnInt((e^(2x)-4)/(x^2-3),x,-1,3)

or $\int_{-1}^{3} ((e^{2x}-4)/(x^2-3))\,dx$

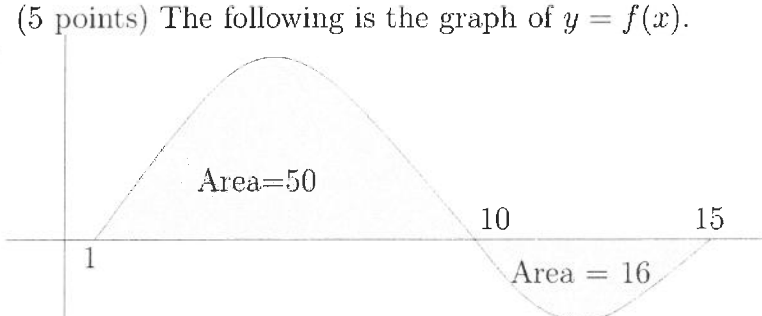
$$\int_{-1}^{3} \frac{e^{2x} - 4}{x^2 + 3}\,dx$$ 19.1477

(1+2fnInt(x^2,x,0,3))/fnInt(2^x,x,0,3)

or $(1 + 2\int_0^3 x^2\,dx)/\int_0^3 2^x\,dx$

$$\frac{1 + 2\int_0^3 x^2\,dx}{\int_0^3 2^x\,dx} =$$ 1.8814

5. (5 points) The following is the graph of $y = f(x)$.



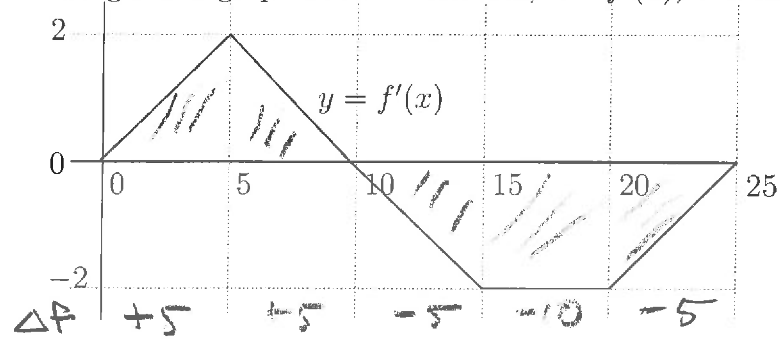
Compute the following

$\int_{10}^{15} f(x)\,dx =$ −16

$\int_{1}^{15} f(x)\,dx =$ 56−16 = 34

6. (10 points) The following is the graph of the derivative, $u = f'(x)$, of a function $y = f(x)$.

Each box is $2 \times 5 = 10$



If $f(0) = -8$ fill in the following table:

x	0	5	10	15	20	25
$f(x)$	−8	−3	2	−3	−13	−18

+5 +5 −5 −10 −5

7. (15 points) Let $r(t)$ be the rate in lbs per week that a baby elephant is growing as a function of t, the number of weeks since it was born. Assume

$$\int_0^4 r(t)\,dt = 75.$$

(a) In this equation

What are the units of 4 weeks

What are the units of 75 lbs

(b) If the baby elephant weights 200 at birth how much does it weight when it is 4 weeks old?

$W(t)$ = weight after t weeks

Its weight is 275 lbs.

$$W(4) = W(0) + \int_0^4 W'(t)\,dt$$
$$= W(0) + \int_0^4 r(t)\,dt$$
$$= 200 + 75 = 275$$

8. (5 points) If $f'(x) = 5(1.3)^x$ and $f(4) = 5$ find the following and explain how you got the answers.

$f(6) =$ 42.5567 $\qquad\qquad f(2) =$ −17.2229

How did you get the answers?

$$f(6) = f(4) + \int_4^6 f'(x)\,dx$$
$$= 5 + \int_4^6 5(1.3)^x\,dx$$
$$= 42.5567$$

$$f(2) = f(4) + \int_4^2 f'(x)\,dx$$
$$= 5 + \int_4^2 5(1.3)^x\,dx$$
$$= -17.2229$$

9. (5 points) If $\int_1^3 f(x)\,dx = 5$ and $\int_1^3 g(x) = 7$ what are

$$\int_1^3 4f(x)\,dx = 4\int_1^3 f(x)\,dx = 4\cdot 5 = 20$$

$\int_1^3 4f(x)\,dx =$ 20

$\int_1^3 (f(x) + g(x))\,dx =$ 12

$$\int_1^3 (f(x) + g(x))\,dx$$
$$= \int_1^3 f(x)\,dx + \int_1^3 g(x)\,dx$$
$$= 5 + 7 = 12$$

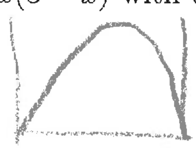
10. (10 points) (a) Plot $y = x(3 - x)$ with $0 \le x \le 3$ and draw the graph here:

\Y1= X(3−X)

Xmin = 0

Xmax = 3

ZoomFit



(b) What is the area between the graph of $y = x(3 - x)$ and the x-axis for $0 \le x \le 3$.

2nd calc 7: ∫f(x)dx

Lower Limit X=0

Upper Limit X=3

∫f(x)dx = 4.5

The area is 4.5

11. 5 free points. ***Have a good thanksgiving.***