Mathematics 172 Homework, August 23, 2019.

1. This problem is preemptive as I usual get ask some version of this several time during a semester. As the syllabus says, if you miss a test, then your score on the test is 80% of the average of your scores on the other tests including the final. It you get 85 on the first test, 90 on the second, miss the third, and then get 136 (out of 150) on the final, then what is your score on the third exam? *Solution:* First the average on the tests you did take is

$$\frac{85+90+136}{1+1+1.5} = 88.857$$

(The reason for the 1.5 is that the final has 1.5 as many points possible as one of the midterm exams.) So the score on the missed exam is

$$80\% \text{ of } 88.857 = (.8)(88.857) = 70.09 \text{ which rounds off to } 70.$$

2. In class some of you sounded like you would like some problems on reviewing taking derivatives. Here are some.

(a) $A(t) = 5e^t$, *Solution:* $A'(t) = 5e^t$.

(b) $P(t) = 13e^{.5t}$, *Solution:* $P'(t) = 6.5e^{.5t}$.

(c) $N(t) = t^5e^{3t}$, *Solution:* Use the product rule:

$$\begin{aligned} N'(t) &= (t^5)'e^{3t} + (t^5)\left(e^{3t}\right)' \\ &= 5t^4e^{3t} + 3t^5e^{3t} \\ &= (5t^4 + 3t^5)e^{3t}. \end{aligned}$$

(d) If $N' = 5N$ show that $u(t) = e^{-5t}N$ satisfies $u' = 0$.*Solution:* Again use the product rule:

$$\begin{aligned} u'(t) &= \left(e^{-5t}\right)' N + e^{5t}N' \\ &= (-5)e^{-5t}N + e^{5t}(5N) && \text{(Using that } N' = 5N\text{)} \\ &= -5e^{-5t}N + 5e^{5t}N \\ &= 0. \end{aligned}$$

Remark: This shows that u is constant, say

$$u = e^{-5t}N = c$$

where c is a constant. Solving for N gives

$$N = \frac{c}{e^{-5t}} = ce^{5t}.$$

Letting $t = 0$ gives

$$N(0) = ce^{5(0)} = ce^0 = c.$$

and therefore

$$N(t) = N(0)e^{5t}.$$

We saw in class that the solution to the rate equation (also called a differential equation)

$$y' = ry$$

where r is a constant is

$$y = y(0)e^{rt}.$$

Note that this rate equation can also be written as

$$\frac{dy}{dt} = ry.$$

Also there is nothing special about the use of the variable y. Thus

$$\frac{dP}{dt} = .15P, \quad P(0) = 75$$

has the solution

$$P(t) = P(0)e^{rt} = 75e^{.15t}.$$

3. Find the solutions to the following:

(a) $P'(t) = 1.2P(t)$, $P(0) = 51$. *Solution:* $P(t) = 51e^{1.2t}$.

(b) $N'(t) = -.15N(t)$, $N(0) = 10.3$. *Solution:* $N(t) = 10.3e^{-.15t}$.

Here is an example of something a bit more complicated. We wish to solve

$$P'(t) = rP(t), \quad P(0) = 400, \quad P(2) = 412.$$

In this case r is unknown. We know that

$$P(t) = P(0)e^{rt} = 400e^{rt}.$$

We get anther equation

$$P(2) = 400e^{2r} = 412.$$

We can solve this to get

$$r = \ln(412/400)/2 = 0.01478$$

and thus

$$P(t) = 400e^{.1478t}$$

4. Solve the following:

(a) $P'(t) = rP(t)$, $P(0) = 51$, $P(3) = 62$. Answer: $P(t) = 51e^{.0651t}$.

(b) $N'(t) = aN(t)$, $A(0) = 97$, $A(10) = 85$. (a is a constant.) Answer: $N(t) = 97e^{-.01321t}$.

(c) $Q'(t) = rQ(t)$, $Q(0) = 513$, $Q(1.3) = 520$. Answer: $Q(t) = 513e^{.0104t}$.

5. If $P'(t) = .15P(t)$ and $P(0) = 100$, then

(a) what is $P(5)$?

(b) how long until $P(t) = 500$?

Solution: We know that $P(t) = 100e^{.15t}$. So for part (a) just plug in $t = 5$, that is $P(5) = 100e^{.15(5)} = 211.7$.

For part (b) we need to solve $P(t) = 100e^{.15t} = 500$. The solution is $t = 10.73$.

6. If $N'(t) = -.05N(t)$ and $N(0) = 5.1$ then

(a) what is $N(20)$? Answer: $N(20) = 1.8761$.

(b) how long until $N(t) = .3$? Answer: $t = 56.664$.