Mathematics 172 Homework.

Here we look a bit more at discrete dynamical systems

$$N_t = f(N_{t-1}).$$

We first review dealing with these on the calculator.

Assume that we have a population grows by

$$N_t = 20 - .4N_{t-1}, \qquad N_0 = 2$$

and we wish to compute what happen in the first 50 years.

Set your calculator sf MODE to SEQ and in WINDOW set

nMin=0

nMax=50 (This 50 corresponds to us wanting the first 50 years.)

Then in the Y= window set

nMin=0

\u(n)=20-.4u($n-1$)

u(nMin)=2

Then 2ND TABLE gives

n	u(n)	
0	2	
1	19.2	
2	12.32	
3	15.072	
4	13.971	
5	14.412	
6	14.235	

If we want N_{20}, then we could scroll down the table, but as Emma pointed out in class today we can do 2ND CALC 1:VALUE and let $n = 10$ to get (at the bottom of the screen) that $n = 10$ and Y=14.285714 Thus

$$N_{10} = 14.284426$$

Likewise we find

$$N_{10} = 14.284426$$
$$N_{20} = 14.285714$$
$$N_{30} = 14.285714$$
$$N_{40} = 14.285714$$
$$N_{50} = 14.285714$$

So we see that by time $t = 20$ that N_t has stabilized to the value 14.285714

Problem 1. Do 2ND CALC 1:VALUE and let $n = 51$. Then you get the message ERR: INVALID. Why is this?

Solution. The reason is that we set nMax=50 so the calculator only computed the values $N_0 = u(0), N_1 = u(1), \ldots, N_{50} = u(50)$ and stopped there. If for some reason we needed N_{51}, or N_{75} we could just set nMax=75 or what ever value is big enough. The down side of using a big value of nMax is that computing the extra values take extra time. If you want to experience a longish wait, set nMax=1000 and compute N_{999}. □ □

Problem 2. For the dynamical system we have just been looking at:

$$N_t = 20 - .4n_{t-1}$$

find the equilibrium point.

Solution. To find the equilibrium point solve

$$20 - .4N = N$$

to get

$$N = \frac{20}{1.4} = 14.2857142857$$

which should look familiar from the above. □

Problem 3. For the dynamical system

$$P_t = P_{t-1} + .3P_{t-1}\left(1 = \frac{P_{t-1}}{100}\right)$$

(a) Find the equilibrium points. *Solution.* They are $P_* = 0$ and $P_* = 100$.
(b) If $P_0 = 5$ compute P_1, P_2, P_5, P_{10}, P_{20}, P_{30}, P_{40} and P_{50}. *Solution:*

$$\begin{aligned}
P_1 &= 6.425 \\
P_2 &= 8.228658 \\
P_5 &= 16.773947 \\
P_{10} &= 45.270958 \\
P_{20} &= 95.598480 \\
P_{30} &= 99.867674 \\
P_{40} &= 99.996255 \\
P_{50} &= 99.999894
\end{aligned}$$