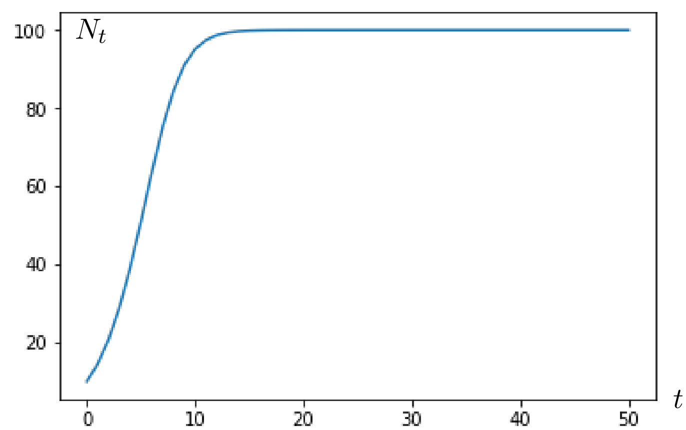
Mathematics 172 Homework.

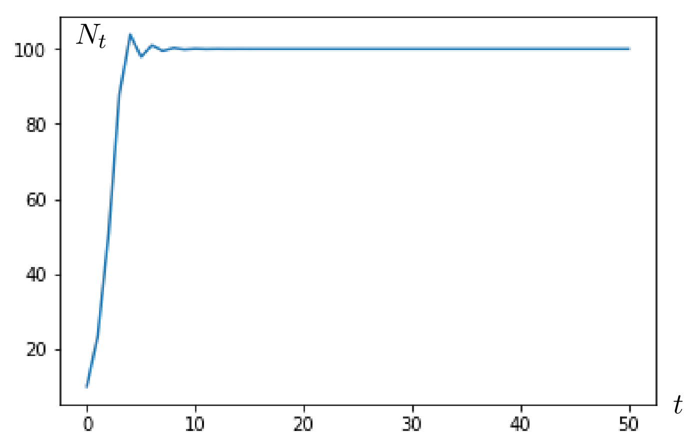
Consider a population that grows with a discrete logistic rule with carrying capacity $K = 100$ and let $N_0 = 10$. That is

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{100}\right)$$

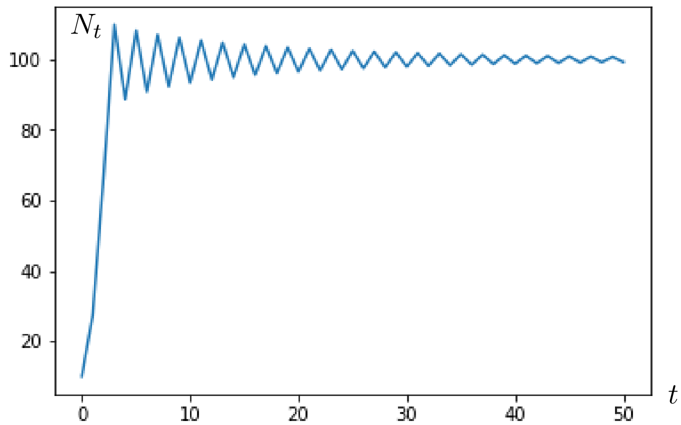
where r is the per capita growth rate. Here we look at what happens if we vary r. We have seen that when $0 < r < 2$ that this is stable. Here are some some graphs (time series) of what happens for some values of r in this range:



Logistic growth with $K = 100$, $r = .5$, $N_0 = 10$

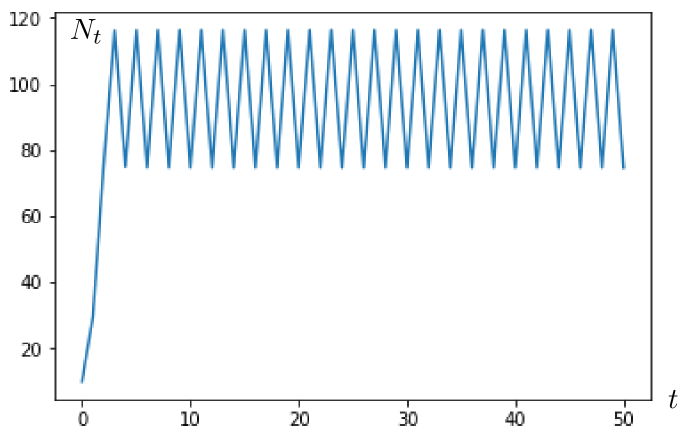


Logistic growth with $K = 100$, $r = 1.5$, $N_0 = 10$

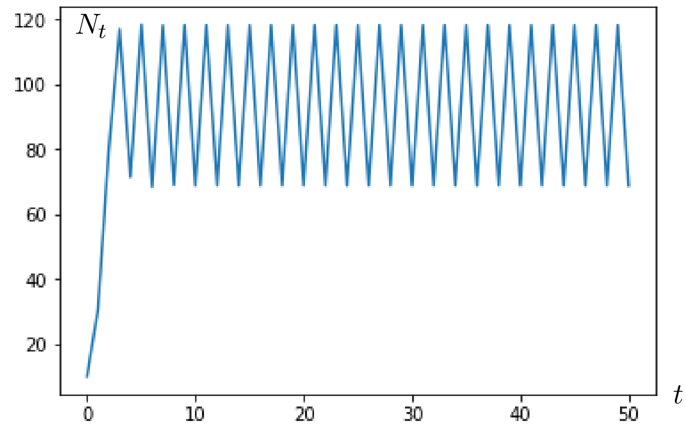


Logistic growth with $K = 100$, $r = 1.95$, $N_0 = 10$

Once r becomes larger than 2, there is no longer a stable equilibrium point and thus N_t can not converge to a single value. But things can still be moderately. Here is what happens for some values of r just a bit bigger than 2.



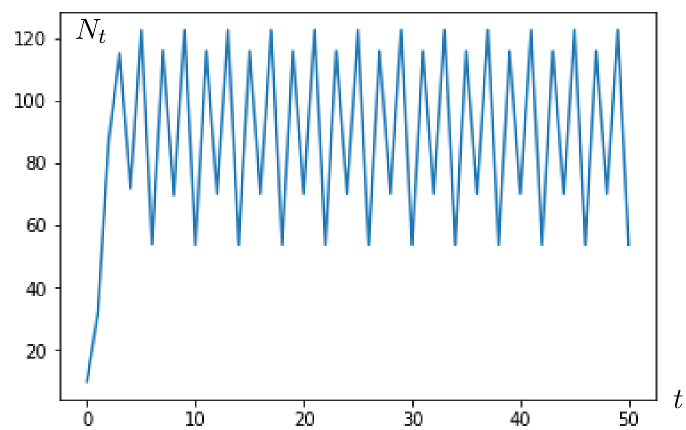
Logistic growth with $K = 100$, $r = 2.2$, $N_0 = 10$



Logistic growth with $K = 100$, $r = 2.3$, $N_0 = 10$

What is happening in these cases is that N_t ends up bouncing back and forth between two values. So in even years it is close to one of the values, and in odd numbered years it is close to the other value.

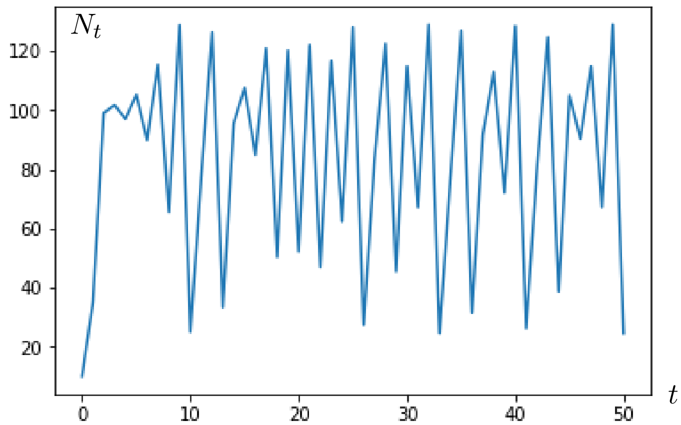
Where r a bit larger, say $r = 2.5$, things get even more wild.



Logistic growth with $K = 100$, $r = 2.5$, $N_0 = 10$

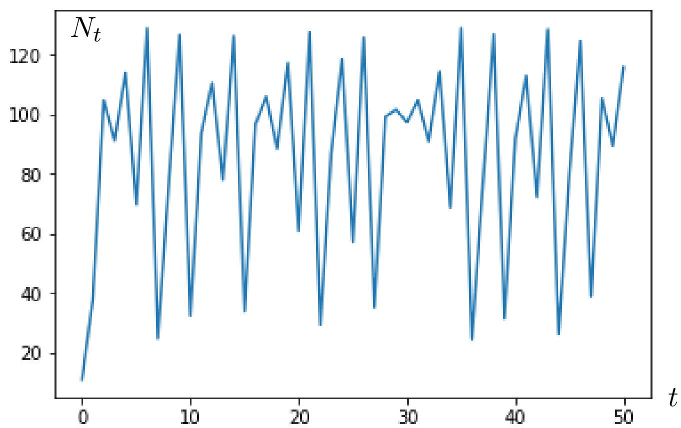
If you look at this for a while you will see there is a pattern. This time N_t bounces cyclically between four values.

If r is increased to $r = 2.8$ then we have achieved chaos:



Logistic growth with $K = 100$, $r = 2.8$, $N_0 = 10$

There is no real pattern in this. Worse yet if we keep K and r the same and just change $N_0 = 10$ to $N_0 = 11$ the picture look entirely different:



Logistic growth with $K = 100$, $r = 2.8$, $N_0 = 11$

Thus this model can be used for short term predictions, but long term predictions are pretty much out of the question.

Problem 1. For the system

$$P_{t+1} = P_t e^{2.8(1-P_t/50)}$$

find the equilibrium points and show they are both unstable. Explain why this system can never stabilize at a single value. □