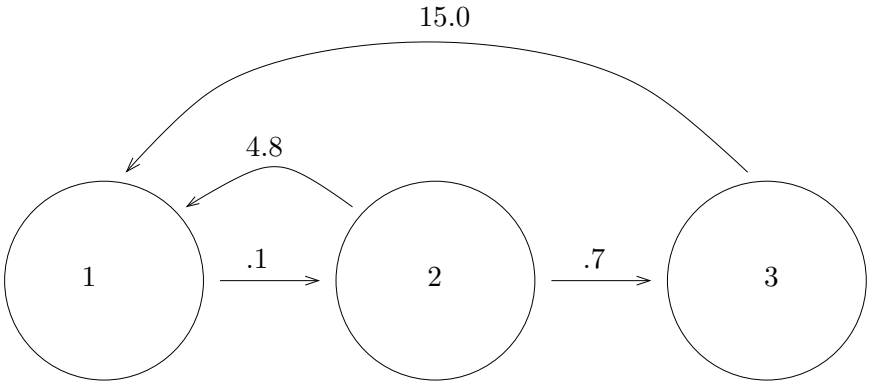
Mathematics 172 Homework, October 7, 2019.

Problem 1. A population of weeds in a back yard has three stages. The first is seedling, the second is juvenile, and the third is adult. The life history of this population is summarized by the loop diagram.



(a) What does the number 4.8 mean?

(b) What does the number .7 mean?

(c) What is the Leslie matrix for this diagram?

(d) If we start with 15 seedlings, 3 juveniles, and 2 adults then how many are in each stage the next year? The second year? The tenth year?

(e) What proportion are in each stage in the 20th year and in the 21st year?

Solution: (a) That on the average each juvenile produces 4.8 offspring that live a year to be seedlings.

(b) That the proportion of juveniles that live to be adults is .7.

(c) The Leslie matrix is

$$L = \begin{bmatrix} 0 & 4.8 & 15.0 \\ .1 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$$

(d) Let $\vec{n}(0)$ be the matrix

$$\vec{n}(0) = \begin{bmatrix} 15 \\ 3 \\ 2 \end{bmatrix}$$

In your calculator enter $[A] = L$ (that is enter the numbers for L into the matrix $[A]$ of your calculator. Enter $[B] = \vec{n}(0)$ with $\vec{n}(0)$ as above. Then your calculator should give

$$L\vec{n}(0) = [A][B] = \begin{bmatrix} 44.4 \\ 1.5 \\ 2.1 \end{bmatrix}$$

Thus next next year there will be 44.4 in stage 1, 1.5 in stage 2, and 2.1 in stage 3.

To find the number in each stage for the second year compute

$$\vec{n}(2) = L^2\vec{n}(0) = [A]^2[B] = \begin{bmatrix} 38.7 \\ 4.44 \\ 1.05 \end{bmatrix}$$

For the tenth year compute

$$\vec{n}(10) = L^{10}\vec{n}(0) = [A]^{10}[B] = \begin{bmatrix} 146.95 \\ 12.00 \\ 6.6 \end{bmatrix}$$

(e) Doing calculations just as in part (d) we find:

$$\vec{n}(20) = \begin{bmatrix} 689.85 \\ 58.81 \\ 35.58 \end{bmatrix} \qquad \text{and} \qquad \vec{n}(21) = \begin{bmatrix} 815.95 \\ 68.98 \\ 41.17 \end{bmatrix}$$

For year $t = 20$ the total number is $N(t) = 689.85 + 58.81 + 35.58 = 784.34$. Therefore the proportion in each class is

$$\text{Proportion in stage 1} = \frac{689.85}{784.34} = 0.879529$$

$$\text{Proportion in stage 2} = \frac{58.81}{784.34} = 0.074991$$

$$\text{Proportion in stage 3} = \frac{35.58}{784.34} = 0.045365$$

For the year $t = 21$ the total number is $N(21) = 815.95 + 68.98 + 41.17 = 926.10$ and this time the proportion in each class is

$$\text{Proportion in stage } 1 = \frac{815.95}{926.10} = 0.881057$$
$$\text{Proportion in stage } 2 = \frac{68.98}{926.10} = 0.074489$$
$$\text{Proportion in stage } 3 = \frac{41.17}{926.10} = 0.0444530$$