## Mathematics 172 Homework.

Here is a summary of what we did in class today. Given a Leslie matrix

$$L = \begin{bmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{bmatrix}$$

the ***growth ratio*** $\lambda$ is the positive solution of the ***Euler-Lotka*** equation

$$\frac{f_1}{\lambda} + \frac{p_1 f_2}{\lambda^2} + \frac{p_1 p_2 f_3}{\lambda^3} = 1$$

and the ***per capita growth rate*** is

$$r = \lambda - 1.$$

Once we have $\lambda$ the stable age distribution is given by the vector

$$\frac{1}{1 + \frac{p_1}{\lambda} + \frac{p_1 p_2}{\lambda^2}} \begin{bmatrix} 1 \\ \frac{p_1}{\lambda} \\ \frac{p_1 p_2}{\lambda^2} \end{bmatrix}.$$

**Problem** 1. A type of biannual plant lives in the desert. Because of the dry conditions small seeds don't have much chance of survival so the plant produces a small number of large seeds. These do not sprout until conditions are good and therefore may say as seeds for several years. Assume the plant has three stages, seed, juvenile, and adult and that the Leslie matrix

$$L = \begin{bmatrix} .81 & 1.02 & 1.2625 \\ .1 & 0 & 0 \\ 0 & .8 & 0 \end{bmatrix}$$

Thus a seed has an 81% chance of staying a seed for the next year.

(a) Find the growth ratio and per capita growth rate. *Solution:* This will be the solution to the equation

$$\frac{.81}{\lambda} + \frac{(.1)(1.02)}{\lambda^2} + \frac{(.1)(.8)(1.2625)}{\lambda^3} = 1$$

The solution to this is

$$\lambda = 1.01$$

and thus

$$r = \lambda - 1 = .01$$

(b) What is the stable age distribution? *Solution:* Plugging into the formula above we get

$$\text{Proportion in stage 1} = .849304$$
$$\text{Proportion in stage 2} = .084089$$
$$\text{Proportion in stage 3} = .066605$$

(c) Assume that 25 adults of the plants are planted on a desert island. How many are in each stage after 2 years and what is the age distribution. *Solution:*

$$\text{Number in stage } 1 = 25.5656$$
$$\text{Number in stage } 2 = 3.1562$$
$$\text{Number in stage } 3 = .0000$$

$$\text{Proportion in stage } 1 = .89010$$
$$\text{Proportion in stage } 2 = .10989$$
$$\text{Proportion in stage } 3 = .0$$

(d) How many in each stage and what is the age distribution on the island after 10 years?

$$\text{Number in stage } 1 = 26.63413$$
$$\text{Number in stage } 2 = 2.63703$$
$$\text{Number in stage } 3 = 2.08900$$

$$\text{Proportion in stage } 1 = .84929$$
$$\text{Proportion in stage } 2 = .08409$$
$$\text{Proportion in stage } 3 = .066613$$

So in this example we have pretty much reached the stable age distribution after 10 years.