Mathematics 172 Homework, August 26, 2019.

We have seen in the last couple of classes that the solution to the initial value problem

$$\frac{dP}{dt} = rP, \qquad P(0) = P_0$$

where r and P_0 are constants is

$$P(t) = P_0 e^{rt}.$$

Recall that the meaning of the derivative $\frac{dP}{dt}$ is the rate of change of P with respect to t. One way to think of this is by the approximation

$$\frac{dP}{dt} \approx \frac{\Delta P}{\Delta t}$$

where $\Delta P = P(t + \Delta t) - P(t)$ is the change in P corresponding to a small change Δt. This shows that the units of the derivative are

$$\text{units of } \frac{dP}{dt} = \frac{\text{units of } P}{\text{units of } t}.$$

1. If $P(t)$ is the number of grams of algae in an aquarium t weeks after it is set up, then what are the units of the derivative $P'(t) = \frac{dP}{dt}$? *Solution:* The units are grams/week or maybe a bit more precisely (grams of algae)/week. $\square$

2. If w is the weight in pounds of a tree that is h feet tall, what are the units of $\frac{dw}{dh}$? *Solution:* The units are (lbs)/(foot). $\square$

In the equation

$$\frac{dP}{dt} = rP$$

we can solve for r to get

$$r = \frac{1}{P}\frac{dP}{dt}.$$

This shows that the units of r are

$$\begin{aligned} \text{units of } r &= \left(\text{units of } \frac{dP}{dt}\right) / (\text{units of } P) \\ &= \big((\text{units of } P)/(\text{units of } t)\big)/(\text{units of } P. \end{aligned}$$

This can be farther reduced to

$$\text{units of } r = \frac{1}{\text{units of } t}$$

but generally this is not the best way to think of the units in the equation $P' = rP$.

3. Let $P(t)$ be the number of kg of duckweed in a pond after t days. Assume that P grows by the rate equation

$$P' = rP$$

What are the units of r and give an interruption of r in terms of these units. *Solution:* The units of r are (kg/day)/kg. This means that each kg of duckweed produces r kg of duckweed each day. □

Duckweed is one of the fastest growing plants.

FIGURE 1. A picture of two types of duckweed from Wikipedia artcle on Lemnoideae (which is the Latin name for Duckweed).

FIGURE 2. Picture of duckweed covering a pond from the same Wikipedia article.

When modeling population growth by the equation $P' = rP$ the constant r has several names in the literature. I have seen the terms ***intrinsic growth rate***, ***per capita growth rate*** and ***relative growth rate*** used. (I will generally reserve "per capita growth rate" for the case of discrete populations, which we will get to later in the term.)

4. The online article *Relative in vitro growth rates of duckweeds (Lemnaceae) the most rapidly growing higher plants* gives the values of r for 13 different species of duckweed in ideal environments. The largest value of r was $r =$ 0.519 (kg/day)/kg.

(a) What is the doubling time for this species. *Solution:* 1.336 days.

(b) If one gram (= .001kg) of this species of duckweed is introduced into a pond on the foot of a duck, then how many days until there are 1,000 kg of duckweed in the pond? *Solution:* 26.619 days. So less than a month.