Mathematics 172 Homework

One model for two species, the x-species and y-species competing for the same resources is

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x\alpha y}{K_1} \right)$$
$$\frac{dy}{dt} = r_2 y \left(\frac{K_2 - \beta x - y}{K_2} \right)$$

where

$$\begin{aligned}
r_1 &= \text{relative growth rate of } x\text{-species} \\
r_2 &= \text{relative growth rate of } y\text{-species} \\
K_1 &= x\text{-species carrying capacity} \\
K_1 &= x\text{-species carrying capacity} \\
K_2 &= y\text{-species carrying capacity} \\
\alpha &= \text{amount of } x\text{-carrying capacity used by a } y \text{ individual} \\
\beta &= \text{amount of } y\text{-carrying capacity used by a } x \text{ individual}
\end{aligned}$$

The solution for these problems are after the last problem. Recall that an ***equilibrium point*** of the system is a point where both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

1. For the system of differential equations

$$\frac{dx}{dt} = .4x \left(\frac{100 - x - .4y}{100} \right)$$
$$\frac{dy}{dt} = .6y \left(\frac{200 - .8x - y}{200} \right)$$

draw the phase plane (which for us is just a fancy term for the first quadrant of the x-y plane) showing

(a) The lines where $\dfrac{dx}{dt} = 0$,

(b) The lines where $\dfrac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Which of the equilibrium points are stable.

(e) Is this competitive coexistence, competitive exclusion, x-species dominates, or y-species dominates.

2. For the system of differential equations

$$\frac{dx}{dt} = .35x\left(\frac{100.0 - x - 1.52y}{100.0}\right)$$
$$\frac{dy}{dt} = .07y\left(\frac{150.0 - 3.75x - y}{150.0}\right)$$

draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Which of the equilibrium points are stable.

(e) Is this competitive coexistence, competitive exclusion, x-species dominates, or y-species dominates.

3. For the system of differential equations

$$\frac{dx}{dt} = .33x\left(\frac{300.0 - x - 0.67y}{300.0}\right)$$
$$\frac{dy}{dt} = .51y\left(\frac{250.0 - 4.17x - y}{250.0}\right)$$

draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Which of the equilibrium points are stable.

(e) Is this competitive coexistence, competitive exclusion, x-species dominates, or y-species dominates.

4. For the system of differential equations

$$\frac{dx}{dt} = .023x\left(\frac{100.0 - x - 2.86y}{100.0}\right)$$
$$\frac{dy}{dt} = .1y\left(\frac{80.0 - 0.40x - y}{80.0}\right)$$

draw the phase plane showing

(a) The lines where $\frac{dx}{dt} = 0$,

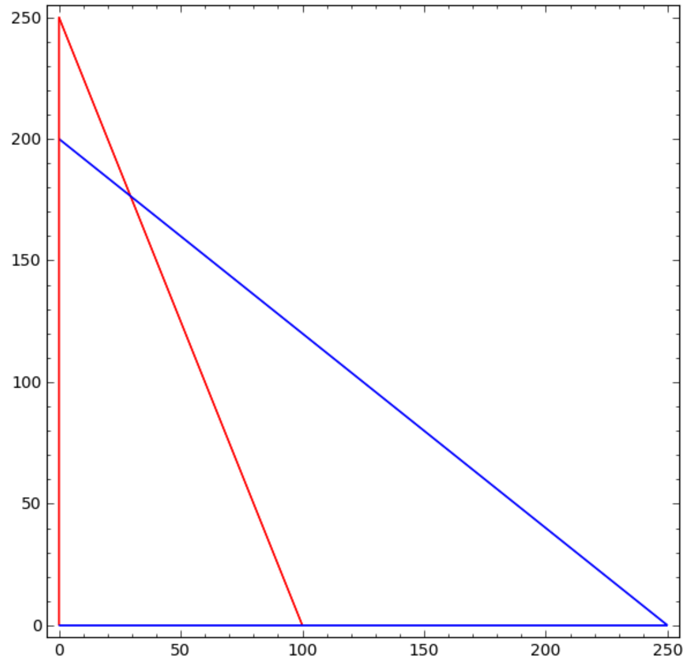
(b) The lines where $\frac{dy}{dt} = 0$,

(c) The coordinates of all the equilibrium points in the first quadrant.

(d) Which of the equilibrium points are stable.

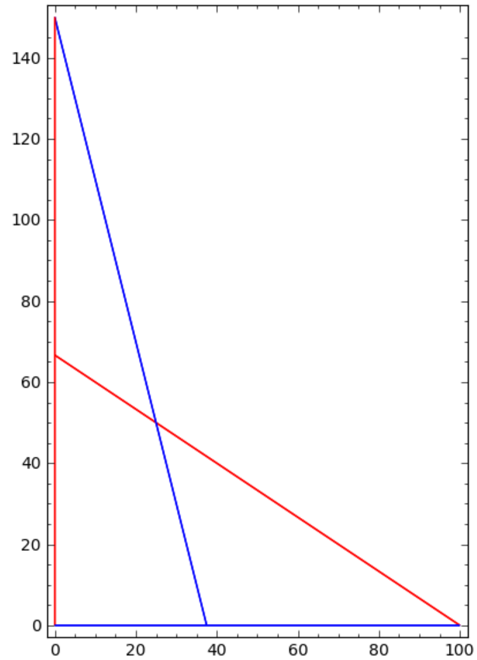
(e) Is this competitive coexistence, competitive exclusion, x-species dominates, or y-species dominates.

Solution to 1:



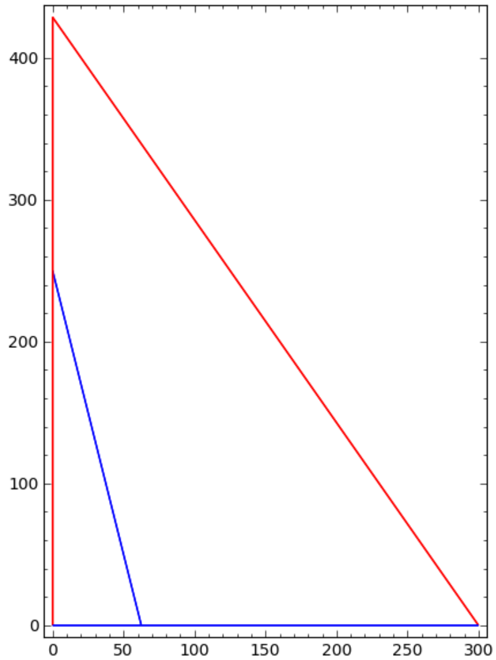
(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(100, 0)$, $(0, 200)$, and $(29.41, 176.5)$. The point $(29.41, 176.5)$ is stable. This is competitive coexistence.

Solution to 2:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0,0)$, $(100,0)$, $(0,150)$, and $(27.2, 47.9)$. The points $(100,0)$ and $(0,150)$ are stable. This is competitive exclusion.

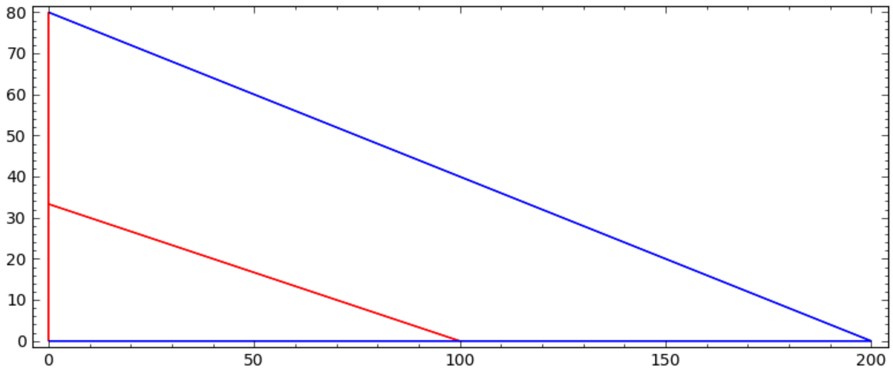
Solution to 3:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0, 0)$, $(300, 0)$, and $(0, 250)$.

The point $(300, 0)$ is stable. This is x-species dominates.

Solution to 4:



(a) The $dx/dt = 0$ lines are in red. (b) The $dy/dx = 0$ lines are in blue. The equilibrium points are $(0,0)$,$(100,0)$, and $(0,80)$. The point $(0,80)$ is stable. This is y-species dominates.