Mathematics 172 Homework.

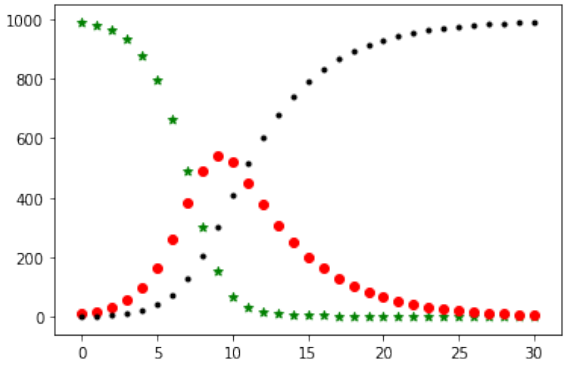
We now have the SIR model

$$\begin{aligned}\Delta S &= -bSI\\ \Delta I &= bSI - kI\\ \Delta R &= kI\end{aligned}$$

into our calculators. Let us start with some experiments the example we started with in class, that is

$$\begin{aligned}b &= .001\\ k &= .2\\ S_0 &= 990\\ I_0 &= 10\\ R_0 &= 0\end{aligned}$$

Plot this for the first 20 days. That is nMin=0 and nMax=30 to get a plot that looks like



Problem 1. Use the trace function on the calculator to find the day on which the maximum number of individuals are infected and what this maximum number is. *Solution:* The maximum occurs on the day $t = 9$ and the maximum number is $I_9 = 543.57$.

Problem 2. Let us look at the example of a commercial hen house with 1,000 hens that are keep together. We consider a virus that has a transmission rate coefficient of

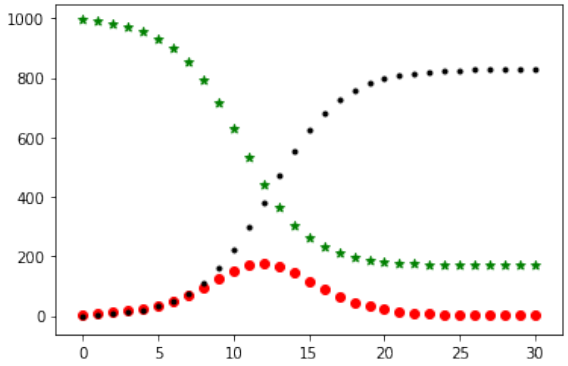
$$b = .001$$

Assume that an infection on the average lasts 2 days. Then we expect the recovery rate is

$$k = \frac{1}{2} = .5$$

Assume that at the 5 infected birds are introduced to the hen house. So $S_0 = 995$, $I_0 = 5$, and $R_0 = 0$. Plot the first 30 days of the infection and find the day when the infection peaks (is at its largest) and the maximum number infected. *Solution:*

The graph is



The infection peaks at day $t = 12$ with a maximum of $I_{12} = 176.14$

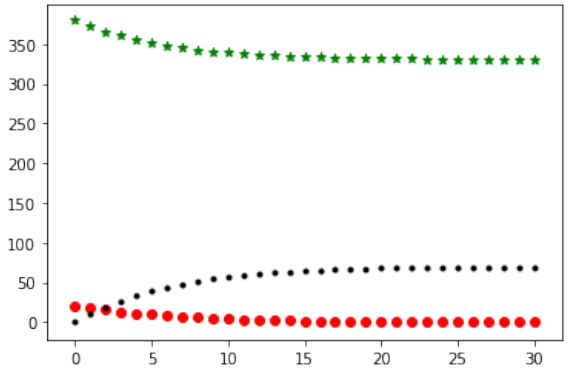
Recall that we showed in class that once S becomes smaller than

$$S = \frac{k}{b}$$

that the infection will start to decrease down to zero. Thus if $S_0 < k/b$ the infection will just die off (by there will still be some unlucky organisms that will get infected). In the hen house example of the last problem we have that this threshold is

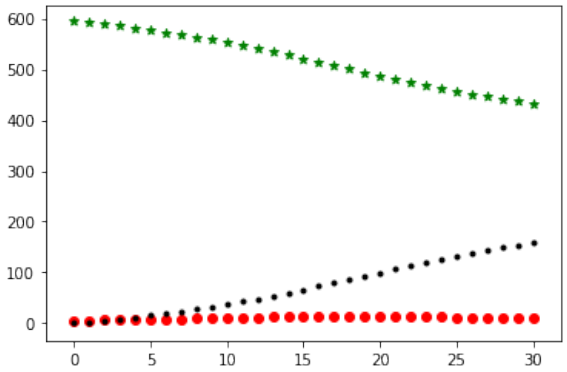
$$\frac{k}{b} = \frac{.5}{.001} = 500.$$

Problem 3. Assume that the hen house of the last problem only holds 400 birds and that 20 are infected at the beginning. That is $S_0 = 380$, $I_0 = 20$ and $R_0 = 0$. Plot what happens over the first 30 days. *Solution:* The graphs are



So the infection dies off rather quickly.

Problem 4. With the same hen house, but this time assuming that it holds 600 birds and that one infected birds is present at the beginning. That is $S_0 = 599$, $I_0 = 5$ and $R_0 = 0$. What happens this time?*Solution:* The graphs for the first 30 days look like



It is a harder to what is happening here, but the infection increasing for the first 19 days, peaking on day $t = 19$ with the maximum number injected being $I_{19} = 13.478$.