Mathematics 172 Homework, September 4, 2019.

Let us look at a rate equation where finding the equilibrium points requires some work. Assume that $N = N(t)$ satisfies

$$\frac{dN}{dt} = -.05N^3 + .7N^2 - 2.1N$$

Find the equilibrium points comes down to solving

$$-.05N^3 + .7N^2 - 2.1N = 0.$$

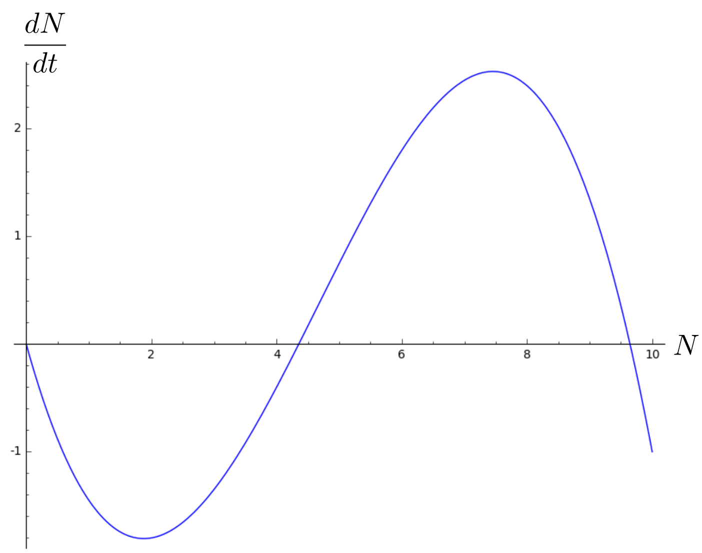
Hopefully that $N = 0$ is one solution is clear. To find others we use the calculator. Use the Y = button and enter

\Y$_1$= -.05 X^3 + .7X^2 -2.1X

Use the WINDOW button and set

Xmin = 0
Xmax = 10

Use the ZOOM button and then 0:ZoomFit to get a graph that looks like:



Here I have labeled the axis N and $\frac{dN}{dt}$ rather than the calculator's variables X and Y. From this graph we see that there are three equilibrium points, and that one of them is 0 (as expected). We first find the root that is just a bit larger than 4. Start by doing 2nd CALC. Choose the option 2:zero. The calculator will then ask you for LeftBound. Move the cursor to the left of the root we are looking for and press ENTER. Then you are ask for a RightBound. Move the cursor to the right of the root and press ENTER. Then you are ask for a GUESS?. Just hit ENTER again. Then you get that

X=4.3542487 Y=0

Therefor $N = 4.3542487$ is an equilibrium point.

1. Do a similar calculation to show that the other equilibrium point is $N = 9.645713$

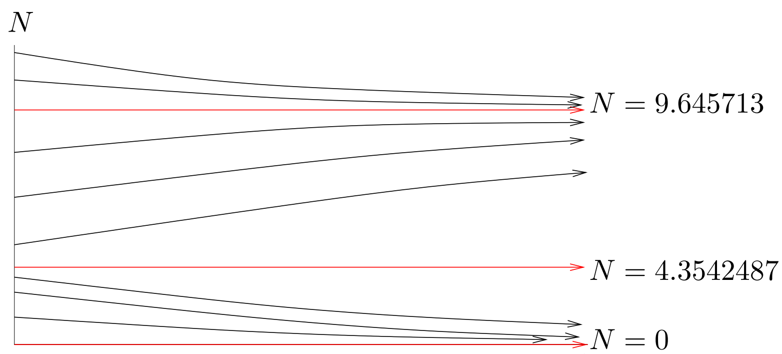
So we now know that the equilibrium points are

$$N = 0,\ 4.3542487,\ 9.645713$$

Looking at the graph above of $\frac{dN}{dt}$ as a function of N we see that

$$\frac{dN}{dt} < 0 \text{ for } 0 < N < 4.3542487 \text{ so } N \searrow$$
$$\frac{dN}{dt} > 0 \text{ for } 4.3542487 < N < 9.645713 \text{ so } N \nearrow$$
$$\frac{dN}{dt} < 0 \text{ for } 9.645713 < N \text{ so } N \searrow$$

Thus if we graph solutions N as a function of t (the ***time series*** of the solutions, the picture will look like:



From this we see that the solutions $N = 0$ and $N = 9.645713$ are stable and $N = 4.3542487$ is unstable.

2. For this equation if $N(0) = 2$ estimate $N(100)$. *Solution:* $N(100) \approx 0$.

3. For this equation if $N(0) = 13$ estimate $N(76)$. *Solution:* $N(76) \approx 9.645713$

Here is a one for you to work on. Let

$$\frac{dP}{dt} = -0.02P^4 + .49P^3 - 3.69P^2 + 8.27P$$

4. Find the equilibrium points. As a hint I will tell you that they are all less than 13 so use Xmin=0 and Xmax=13 when doing the plot. *Solution:* They are (to three decimal places)

$$P = 0, \quad 4.099, \quad 8.418, \quad 11.982$$

5. Make graphs showing the equilibrium solutions and some other solutions to see which of the equilibrium points are stable.

6. What are the stable equilibrium points? *Solution:* The stable points are $P = 4.099$ and $P = 11.982$.

7. What are the unstable equilibrium points? *Solution:* They are $P = 0$ and $P = 8.418$.
8. If $P(0) = 7.3$ estimate $P(100)$. *Solution:* $P(100) \approx 4.099$.
9. If $P(0) = 15$ estimate $P(98.6)$. *Solution:* $P(98.6) \approx 11.982$.