Mathematics 172

Quiz 15 Name: Key

You must show your work to get full credit.

1. An amusement park has as one of its attractions a wishing well. Guests are to throw pennies into the pond to get their wishes. The well has a population of goldfish. Unfortunately copper is poisonous to fins and so each month 15% of the fish die off. The management of the park wish to keep a stable population of 200 gold fish in the pond. At what monthly rate should they stock the wishing well for this to happen?

Let N_t = number of gold fish in year t, and S = stocking rate. Then

$N_{t+1} = N_t - .15N_t + S$

They wish $N_t = 200$ to be an equilibrium point so

$200 = 200 - .15(200) + S$

$S = .15(200) = 30$

The stocking rate is 30 fish/month.

2. For the discrete dynamical system

$$P_{t+1} = P_t e^{3\left(1-\frac{P_t^2}{90}\right)}$$

(a) What are the equilibrium points? *Hint:* Try Xmin=0 and Xmax=10.

\Y1=X e^(3(1-X^2/90))

\Y2=X

Xmin=0

Ymax=10

0: ZoomFit

Equilibrium points are: 0, 9.4868

$P_0 = 0$ is clearly an equm pt.

2nd calc 5: intersect to get $P_* = 9.4868$ is the other.

(b) Explain why for this system the solutions never settle down and converge to any stable values.

At $P_0 = 0$ 2nd calc 6: dy/dx $X=0$ gives $\frac{dy}{dx} = 20.09$ and so $\left|\frac{dy}{dx}\right| > 1$. Thus unstable

At $P_* = 9.4868$ $\frac{dy}{dx} = -5.00$ and thus $\left|\frac{dy}{dx}\right| > 1$. Also unstable

Thus There are no stable equilibrium points, so the system can never converge to a point