Mathematics 172

Quiz 25 **Name: Solution Key.**

You must show your work to get full credit.

We have a population of predators and victims in some environment. Let $P(t)$ be the size of the predator population at time t and $V(t)$ the size of the victim population. The most basic model for the interaction between the two is the ***Lokta-Volterra equations***:

$$\frac{dV}{dt} = rV - aVP$$
$$\frac{dP}{dt} = -mP + bVP.$$

Here r is the relative grow rate of the victim population when no predators are present. The constant m is the mortality rate of the predator population when there is is no prey. The constant a is a measure of how efficient the predator is at reducing the prey population. And b measures how nourishing the prey is to the predator. Let us look at an example. We have an aquarium which has population of single cell algae growing in it. There is a population of Paramecium also living in the aquarium which is feeding off of the algae. Let $V(t)$ be the number of grams of algae in the tank after t days and $P(t)$ then number of grams of Paramecium. Assume that these satisfy

$$\frac{dV}{dt} = .3V - .05VP$$
$$\frac{dP}{dt} = -.8P + .1VP$$

1. What are the rest points of this system?

Solution. We need to solve the system of equations

$$.3V - .05VP = V(.3 - .05P) = 0$$
$$-.8P + .1VP = P(-.8 + .1V) = 0$$

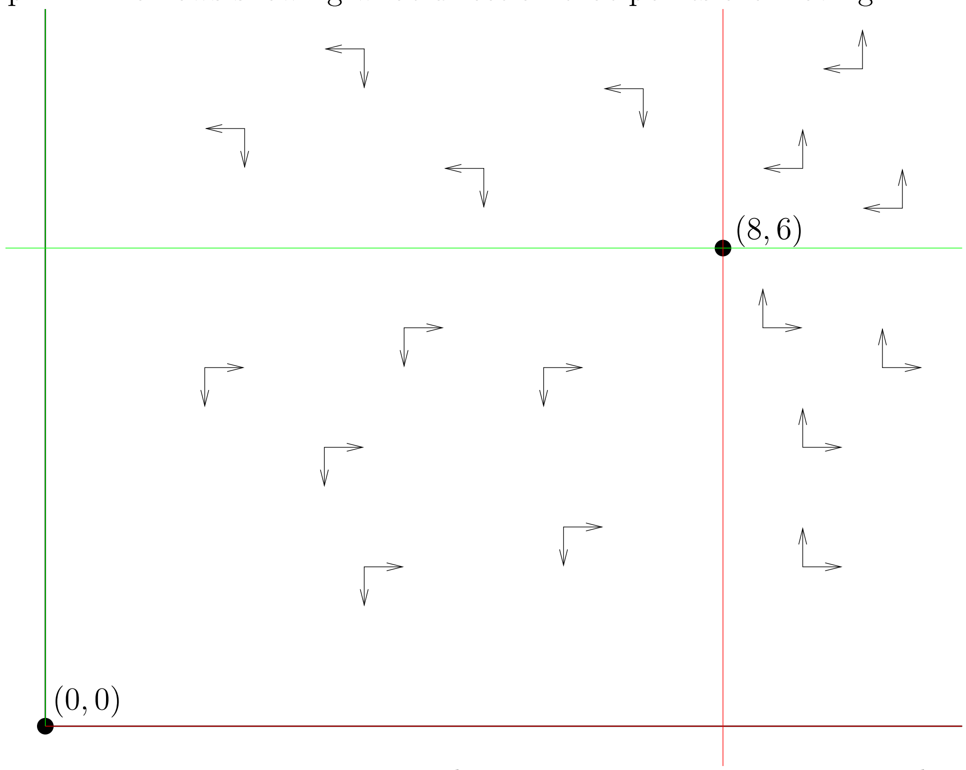
The first of these two equation implies $V = 0$ or $P = \widehat{P} = .3/.05 = 6.0$. The second implies $P = 0$ or $V = \widehat{V} = .8/.1 = 8$. Therefore

The rest points are: $(0,0)$ and $(\widehat{V}, \widehat{P}) = (8,6)$

□

2. Draw in the lines where $\frac{dV}{dt} = 0$ and the lines where $\frac{dP}{dt} = 0$. Use the horizontal (i.e. x) axis for V and the vertical (i.e. y) axis for P.

3. In your graph fill in arrows showing what direction that points are moving.



The rest points are the points ●. The lines $\frac{dV}{dt} = 0$ are in green and the lines $\frac{dP}{dt} = 0$ are in red.