

Quiz 27

Name: key*You must show your work to get full credit.*

Consider the predator-victim system

$$\frac{dV}{dt} = .1V \left(1 - \frac{V}{600} \right) - .01VP$$

$$\frac{dP}{dt} = -.4P + .002VP$$

1. The point $(V, P) = (0, 0)$ is a rest point of this system. Write a sentence or two explaining the biology of this rest point. *If we start with no victims or predators, then the population of both stays at 0.*

2. If there are no predators present, that is if $P = 0$, then the equation for victims becomes

$$\frac{dV}{dt} = .1V \left(1 - \frac{V}{600} \right)$$

which is the logistic equation with intrinsic growth rate $r = .1$ and carrying capacity $K = 600$. Thus $(V, P) = (600, 0)$ is a rest point. Write a sentence or two explaining the biology of this rest point. *If there are no predators, the population of victims stabilizes at its carrying capacity of $K = 600$.*

3. We now look for a rest point with $P \neq 0$ and $V \neq 0$. Using this in the equations $\frac{dV}{dt} = \frac{dP}{dt} = 0$ leads to the system of equations

$$\begin{aligned} (*) \quad .1 \left(1 - \frac{V}{600} \right) - .01P &= 0 \\ -.4 + .002V &= 0. \end{aligned}$$

Solve the second equation for V and then use this in the first equation to find the third rest point.

$$\begin{aligned} .002V &= .4 \\ V &= \frac{.4}{.002} = 200 \end{aligned}$$

The third rest point is $(200, 6.67)$

use this in ()* $.1 \left(1 - \frac{200}{600} \right) - .01P \Rightarrow P = \frac{.1 \left(1 - \frac{1}{3} \right)}{.01} = 6.67$

4. It turns out that this last rest point is stable. Write a sentence or two explaining the biology of this case. *The carrying capacity of V is greater than the famine line of P . So both populations coexist and the populations sizes stabilize at $V = 200$, $P = 6.67$.*

5. Use the back of this paper to draw the V - P phase space and label the rest points, and line $\frac{dV}{dt} = 0$

(which corresponds to the carrying capacity) and the line where $\frac{dP}{dt} = 0$, which corresponds to the famine line for the predator.

