Mathematics 172

Quiz 28 Name: Key

You must show your work to get full credit.

Consider a SIR system

$$\Delta S = -bSI$$
$$\Delta I = bSI - kI$$
$$\Delta R = kI$$

Rewrite the equation for the change in I as

$$\Delta I = I(bS - k) = bI\left(S - \frac{k}{b}\right).$$

1. Explain why S is decreasing.

$\Delta S = -bSI < 0$ so the change is S is always decreasing.

2. Explain why $S > \frac{k}{b}$ implies I is increasing.

Then $\Delta I = bI(S - \frac{k}{b}) > 0$, i.e. the change in I is positive.
So I is increasing

3. Explain why $S < \frac{k}{b}$ implies I is decreasing.

$\Delta I = bI(S - \frac{k}{b}) < 0$ is negative so I is decreasing.

4. Assume that we have a population such that the number of susceptible individuals is less than $\frac{k}{b}$ and there are no injected individuals. Explain why adding a few infected individuals will not start an epidemic.

If $S_0 < \frac{k}{b}$, then, as S is decreasing, $S < \frac{k}{b}$ in the future. This means that I is always decreasing. Thus the infections dies off. (Decreases to 0).