Mathematics 172

Quiz 34 Name: Key

You must show your work to get full credit.

1. If a quarter of pound of duckweed is put into a large pound and the amount of duckweed doubles every 4 days, then how long until there is a 3 tons (6,000 pounds) of duckweed?

Let $W(t)$ = weight of duckweed in pound after t days. Then

$W(t) = W(0)\lambda^t = .25\lambda^t$

$W(4) = .25\lambda^4 = 2(.25)$

$\lambda^4 = 2$

$\lambda = 2^{\frac{1}{4}} = 1.1892$

so

$W(t) = .25(1.1892)$

Time to 3 tons: 58.20 days

solve

$W(t) = .25(1.1892)^t = 6000$

$(1.1892)^t = \frac{6000}{.25}$

$t\ln(1.1892) = \ln(6000/.25)$

$t = \ln(6000/.25)/\ln(1.1892)$

$= 58.20$

2. If $A'(t) = .2A(t)$ and $A(0) = 75$

(a) Give a formula for $A(t)$. $A(t) =$ $75e^{.2t}$

$A(t) = A(0)e^{rt}$

$= 75e^{.2t}$

solve $A(t) = 75e^{.2t} = 2(75)$

$e^{.2t} = 2$

$.2t = \ln(2)$

$t = \frac{\ln(2)}{.2} = 3.466$

(b) What is the doubling time? Doubling time is: 3.466

3. A population of dandelions grows in a yard with a discrete logistic growth rate of 1.6 dandelions/dandelion and a carrying capacity of 700 dandelions. Let N_t be the number of dandelions in year t.

(a) What is formula for N_{t+1} in terms of N_t?

$N_{t+1} =$ $N_t + 1.6N_t\left(1 - \frac{N_t}{700}\right)$

(b) If $N_0 = 50$ what are N_1 and N_2?

$N_1 =$ 124.25 $N_2 =$ 287.84

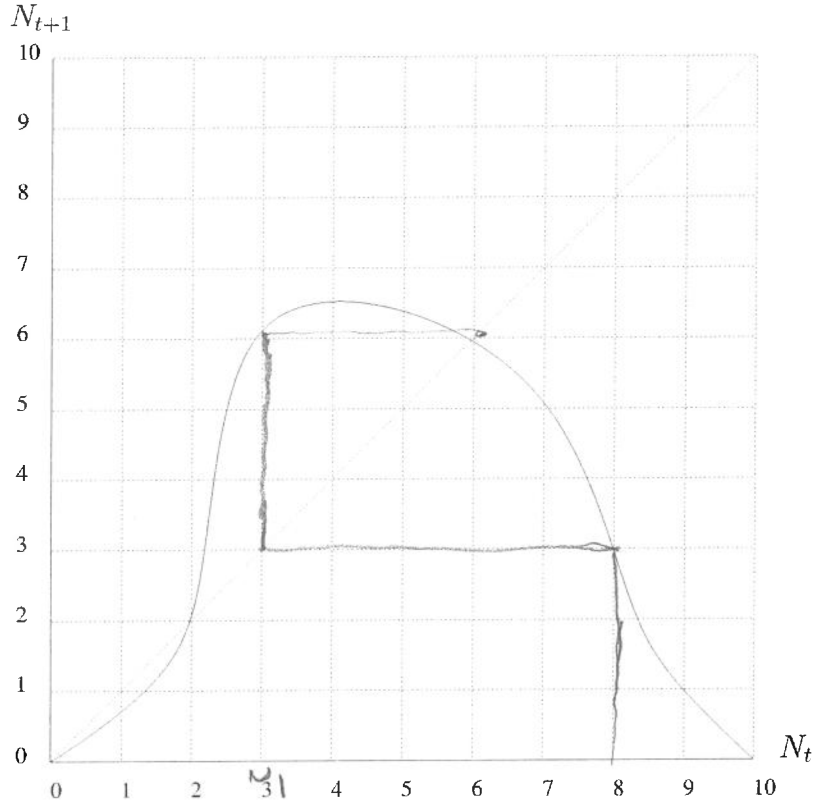
$50 + 1.6(50)\left(1 - \frac{50}{700}\right)$

$N_2 = N_1 + 1.6N_1\left(1 - \frac{N_1}{700}\right)$ with $N_1 = 124.25$

(c) If $N_0 = 50$ estimate N_{78}. $N_{78} \approx$ 700

As $0 < r = 1.6 < 2$ the population stabilizes at the carrying capacity of $K = 700$

4. The following is the graph of a discrete dynamical system that models the number of kilograms of algae in a pond from year to year.



(a) What are the equilibrium points?

Equilibrium points are: 0, 2, 6

(b) What are the stable equilibrium points? Stable points are: 0, 6

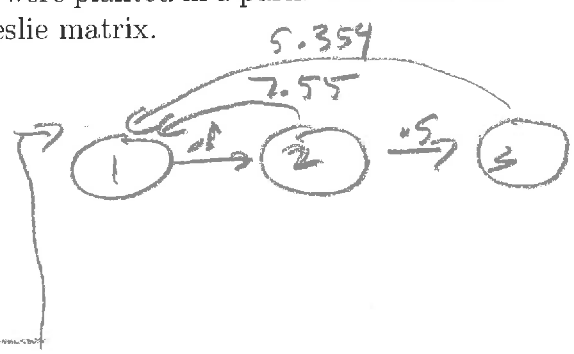
(c) If $N_0 = 8$ estimate the following:

$N_1 \approx$ 3 $N_2 \approx$ 6.1

(d) If $N_1 = 9$ estimate N_{100}. Stabilizes at $N_t = 6$ $N_{100} \approx$ 6

5. A population of foxglove (a biennial plant) plant has three stages: seedlings, juveniles, and adults. This population of foxglove started when 200 seedlings were planted in a park. The relations betweens the three stages are summarized by the following Leslie matrix.

$$L = \begin{bmatrix} 0 & 7.55 & 5.354 \\ .1 & 0 & 0 \\ 0 & .5 & 0 \end{bmatrix}$$



(a) Draw the loop diagram;

(b) Give the meaning of the following numbers:

(i) 7.55. = The average number of stage 1 offspring to a stage 2 individual.

(ii) .5 = proportion of stage 2 individuals that live to stage 3

(c) What proportion of seedlings become adults. Proportion is .05

(.1)(.5) = .05

(d) How many plants are in each stage the first year after planting the seedlings?

Stage 1 0 Stage 2 20 Stage 3 0

(e) How many plants are in each stage 25 years after planting the seedlings?

Stage 1 113.43 Stage 2 11.23 Stage 3 5.56

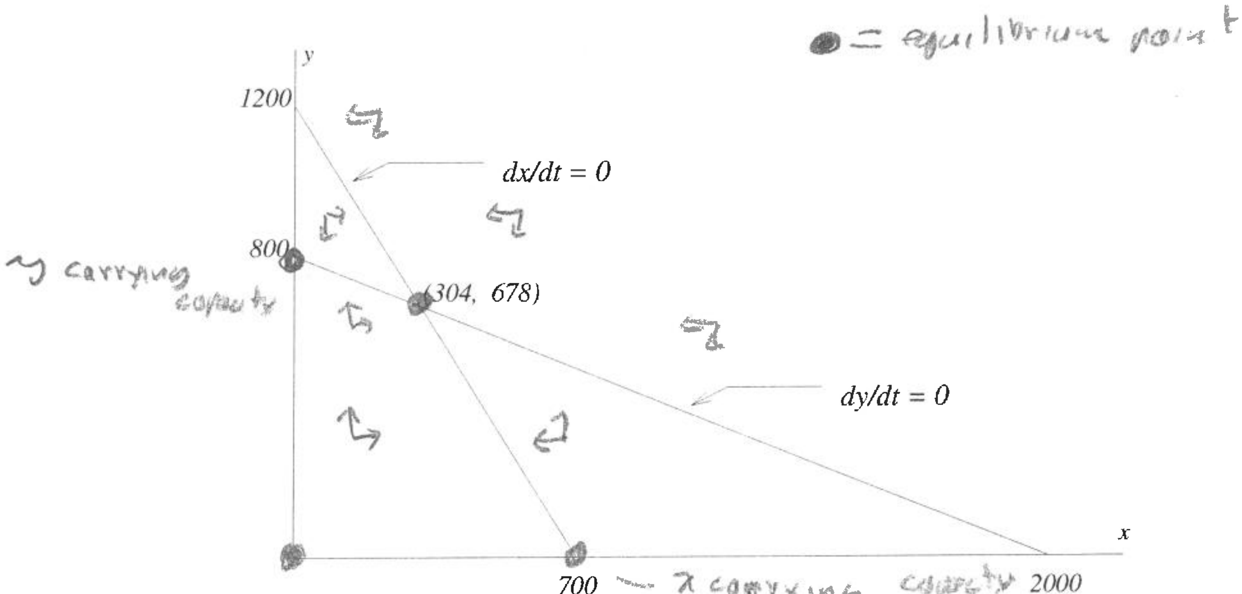
(f) What proportions of the plant population is in each stage 25 years after planting the seedlings?

Stage 1 .870 $\frac{113.2}{130.32}$ Stage 2 .086 $\frac{11.23}{130.32}$ Stage 3 .043 $\frac{5.56}{130.32}$

N = total = 113.43 + 11.23 + 5.56 = 130.32

6. A system of competing species is governed by the equations

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x - \alpha y}{K_1}\right) \qquad \frac{dy}{dt} = r_2 x \left(\frac{K_2 - \beta x - y}{K_2}\right)$$



(a) What is x carrying capacity? ____________

(b) What are the stable equilibrium points? (304, 678)

(c) What are the unstable equilibrium points? (0,0), (700, 0), (0, 800)

(d) If $x(0) = 720$ and $y(0) = 15$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 304 $y(100) \approx$ 678

The point will move up to the stable point (304, 678)

(e) If $x(0) = 720$ and $y(0) = 0$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 700 $y(100) \approx$ 0

This time there are no y-species, so $y(t) = 0$ all t.

Then $\frac{dx}{dt} = r_1 x\left(\frac{K_1 - x}{K_1}\right)$ so $x \to 700$

(f) Circle the one that applies: x species dominates, x species dominates, competitive coexistence, competitive exclusion.

7. A cell has a volume of $V = 6.0 \times 10^{-6}\text{mm}^3$ and a surface area of $A = 8.1 \times 10^{-3}\text{mm}^2$. Assume that oxygen passes through the cell membrane at a rate of $.4(\text{mg}/\text{mm}^2)/\text{hr}$. Assume that the cell needs $65(\text{mg}/\text{mm}^3)/\text{hr}$ of oxygen to survive. Then how many times can it be magnified before it dies from lack of oxygen?

Magnification factor is $\lambda =$ 8.308

Let λ = magnification factor.

magnified volume $= V_\lambda = 6 \times 10^{-6} \lambda^3$ mm^3

magnified Area $= A_\lambda = 8.1 \times 10^{-3} \lambda^2$ mm^2

Total O_2/hour $= A_\lambda \times$ rate

$= 8.1 \times 10^{-3} \lambda^2 \times (.4) = .00324\lambda^2$ mg/hr

Total O_2 (mg/mm^3)/hr $= \frac{.00324\lambda^2}{6 \times 10^{-6}\lambda^3} = \frac{540}{\lambda}$ (mg/mm^3)/hr

solve $\frac{540}{\lambda} = 65$

$\lambda = \frac{540}{65} = 8.308$