Mathematics 172 Test 1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (15 points) The zoo sets up a large tank for South American fish. At some point they add some plants to the tank and the plants have 40 grams of brown algae on them. After 3 weeks there is 100 grams of the algae in the tank.

(a) What is the intrinsic growth rate of the algae? Include units in your answer.

$P(t) = P_0 e^{rt} = 40 e^{rt}$
$P(3) = 40 e^{3r} = 100$
$e^{3r} = \frac{100}{40}$

$3r = \ln(\frac{100}{40})$
$r = \frac{1}{3} \ln(\frac{100}{40})$
$= .3054$

$r =$ $.3054$

(b) If $P(t)$ is the number of grams of brown algae in the tank t weeks after the plants are added, then give a formula for $P(t)$.

$P(t) =$ $40 e^{.3054t}$

(c) How long until there is 500 grams of brown algae in the tank?

Time to 500 grams is. 8.27 weeks

Solve
$40 e^{.3054t} = 500$
$e^{.3054t} = \frac{500}{40}$
$.3054t = \ln(500/40)$
$t = \frac{\ln(500/40)}{.3054} = 8.27$

2. (10 points) Water hyacinth is sometimes used a method to reduce excesses of nitrites and nitrates in polluted water. The mangers of a a badly polluted pond find that the water is of such bad quality that water hyacinth has an intrinsic growth rate of $-.05$ (kg/week)/kg. They wish to keep a stable population of a metric ton (that is 1,000 kg) of water hyacinth in the pond. At what rate should they stock it?

$P(t) =$ kg water hyacinth in week t.

The stocking rate is 50 kg/week

S = stocking rate
$P' = -.05P + S$
$P = 1{,}000$ is equilibrium points so
$0 = -.05(1000) + S$
$S = .05(1000) = 50$

3. (20 points) Algae is growing in a large bucket of water. Let $W(t)$ be the weight in grams of the algae in the bucket after t days. Assume that W satisfies the rate equation

$$\frac{dW}{dt} = .05W\left(1 - \frac{W}{60}\right)\left(\frac{W}{10} - 1\right).$$

(a) If $W(4) = 40$, what is $W'(4)$? $W'(4) =$ 2

$W'(4) = .05(40)\left(1 - \frac{40}{60}\right)\left(\frac{40}{10} - 1\right)$
$= 2$

(b) What are the equilibrium points of this equation?

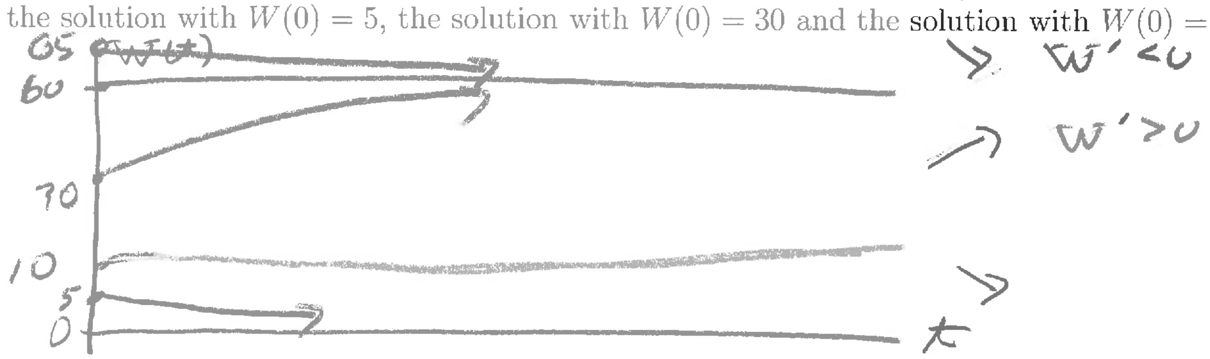
The equilibrium points are: 0, 10, 60

Solve

$\frac{dW}{dt} = .05\,W\left(1 - \frac{W}{60}\right)\left(\frac{W}{10} - 1\right) = 0$

to get $W = 0, 60, 10$

(c) Draw a picture (graph of W as a function of t) which shows the equilibrium solutions and also the solution with $W(0) = 5$, the solution with $W(0) = 30$ and the solution with $W(0) = 65$.



(d) Which of the equilibrium points are stable?

The stable points are: 0, 60

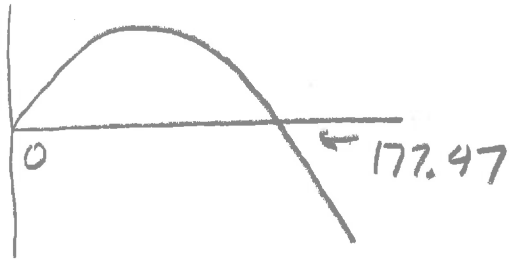
(e) If $W(0) = 5$, estimate $W(100)$. $W(100) \approx$ 0

(f) If $W(0) = 30$, estimate $W(93)$. $W(93) \approx$ 60

4. (15 points) For the rate equation

$$\frac{dy}{dt} = .5y\left(1 - \frac{y^{1.2}}{500}\right)$$

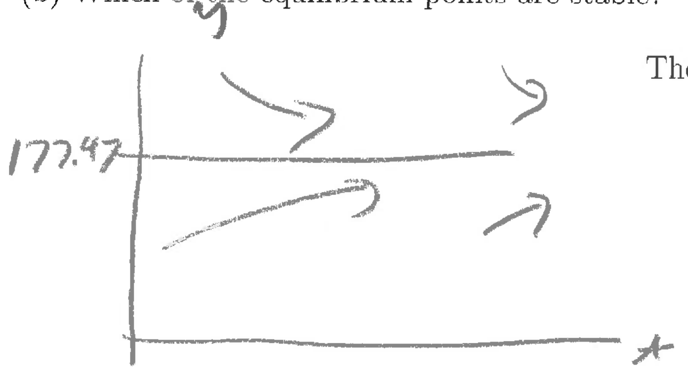
(a) Use your calculator to find the equilibrium points *Hint:* Using Xmin=0 and Xmax=200 is a good choice.



Equilibrium points are $\underline{0,\ 177.47}$

$\backslash Y1 = .5x(1 - x \wedge 1.2/500)$

(b) Which of the equilibrium points are stable?



The stable points are $\underline{177.47}$

(c) If $y(0) = 150$ compute $y'(150)$ and use this to estimate $y(0.1)$.

$y'(0) =$ __________ $y(0.1) \approx \underline{151.37}$

$y'(0) = .5(150)\left(1 - \frac{150^{1.2}}{500}\right) = 13.70$

I used 2nd calc 1: value $x = 150$

$y(0.1) \approx y(0) + y'(0)(.1)$

$= 150 + (13.70)(.1)$

$= 151.37$

5. (10 points) A population of annual plants is introduced to an island. Originally there are 15 of the plants. After 4 years there are 75. What are the growth ratio and per capita growth rate?

$\lambda = \underline{1.495}$ $r = \underline{.495}$

$P_t = P_0\lambda^t = 15\lambda^t$

$P_4 = 15\lambda^4 = 75$

$\lambda^4 = \left(\frac{75}{15}\right)$

$\lambda = \left(\frac{75}{15}\right)^{\frac{1}{4}} = 1.495$

$r = \lambda - 1 = .495$

6. (15 points) A population of duckweed is growing logistically in a pond with an intrinsic growth rate of $r = 2.5$ (lbs/lb)/week and a carrying capacity of $K = 100$ pounds.

(a) The owner of the pond wishes to get rid of the duckweed. What is the least rate she can harvest it so that it eventually is eradicated. Write a sentence or two and include a picture explaining how you got the answer.

Logistic equation is

Harvesting rate is (include units) 62.5 lbs/week.

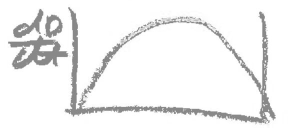
$\frac{dP}{dt} = rP(1-\frac{P}{K})$

$\frac{dP}{dt} = 2.5P(1-\frac{P}{100})$

\Y1 = 2.5X(1-X/100)

Xmin = 0

Xmax = 100



Max is at X

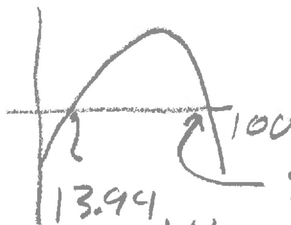
X = P = 50

$Y = \frac{dP}{dt} = 62.5$

(b) What happens to the duckweed population if it is harvested at the rate of 30 lbs/week?

The new equation is

$\frac{dP}{dt} = 2.5P(1-\frac{P}{100}) - 30$



Population stabilizes at 86.06 lbs.

7. (15 points) A population of annual cicadas are living on a small island. Let N_t be the size of the size of the population in year t and assume that

$$N_{t+1} = N_t e^{1.2(1-N_t/500)}.$$

(a) If $N_{10} = 450$ what are N_{11} and N_{12}? Give your answer to two decimal places.

$N_{11} =$ 507.37

\Y1 = Xe^(1.2(1-X/500))

Xmin = 0

Xmax = 600

Zoom Fit

2nd calc 1:value X = 450

Y = 507.37

$N_{12} =$ 498.47

2nd calc 1:value

X = ALPHA Y

Y = 498.47

(b) What are the equilibrium points of this system? (This can be done without the calculator, but if you use it I suggest letting Xmin=0 and Xmax=600)

\Y1 = Xe^(1.2(1 - X/500))

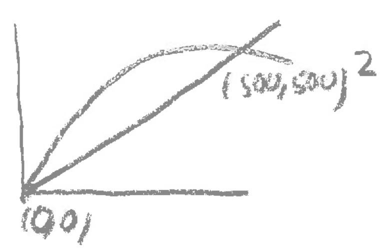
\Y2 = X

Equilibrium points are 0, 500

Xmin = 0

Xmax = 600

Zoom Fit



2nd calc 5:intersect

500