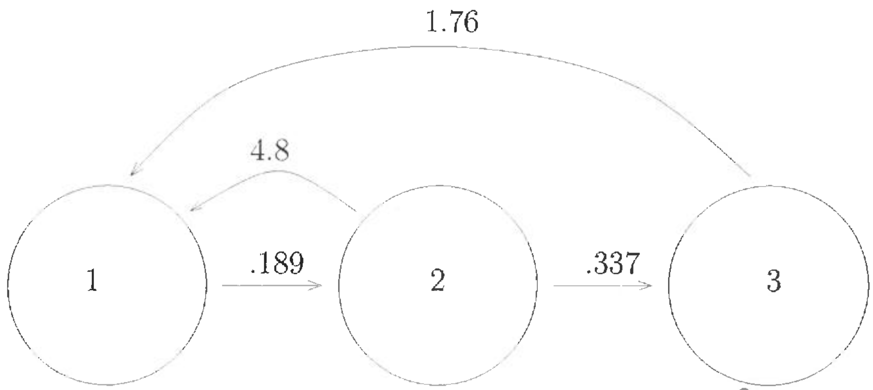
Mathematics 172 Test 2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (20 points) Some species killifish breed just once a year and live to be a most two years old. We look at three life stages. Stage 1: hatchling, Stage 2: one year old, and Stage 2: two years old. The life history of a population of killifish living in the ponds, puddles, and streams of a small island is summarized by the loop diagram:



(a) What is the Leslie matrix?

$$L = \begin{bmatrix} 0 & 4.8 & 1.76 \\ .189 & 0 & 0 \\ 0 & .337 & 0 \end{bmatrix}$$

(b) What does the number 1.76 mean?

It is the average number of offspring to a stage 3 individual that live to stage 1.

(c) What does the number .189 mean?

It is the proportion of stage 1 individuals that live to stage 2

(d) What proportion of the hatchlings live to be two years old?

It takes 2 steps to go from (1) to (3) The proportion is $(.189)(.337) = .063693$

(e) If this year there are 501 harchlings, 82 one year olds, and 20 two year olds, then after 20 years how many are in each stage and what proportion are in each stage?

Number in each stage:

Stage 1 556.94 Stage 2 120.89 Stage 3 34.90

$[A] = L$ $\quad B = \begin{bmatrix} 501 \\ 82 \\ 20 \end{bmatrix} = \vec{n}(0)$ $\quad \vec{n}(20) = [A]^{20}[B] = \begin{bmatrix} 556.94 \\ 120.89 \\ 34.90 \end{bmatrix}$

Total $= N = 556.94 + 102.89 + 34.90 = 694.73$

Proportion in each stage:

Stage 1 .802 $\frac{556.94}{694.73}$ Stage 2 .148 $\frac{120.89}{694.73}$ Stage 3 .050 $\frac{34.90}{694.73}$

2. (20 points) Anther population of killifish has the same three stages. It has reached its stable age distribution and has a growth ratio of $\lambda = 1.02$.

(a) Explain what it means that it means for the population to have reached its stable age distribution.

The proportion in each stage stays the same from year to year.

(b) What is the per capita growth rate? $r =$ $\lambda - 1 = .02$

(c) If this year there are 798 in Stage 1, 245 in Stage 2, and 237 in Stage 3, how many are in each stage next year?

Number in each stage:

Stage 1 $813.96 = (1.02)(798)$ Stage 2 $249.9 = (1.02)(245)$ Stage 3 $241.74 = (1.02)(237)$

Just multiple the number in stage by $\lambda = 1.02$ to get the number for the next year

3. (20 points) A population of grasshoppers lives in an isolated field. Let N_t be the size of the population measured in 1,000's of grasshoppers and assume

$$N_{t+1} = \frac{2.1N_t}{1 + .10N_t^2}$$

(a) If $N_0 = 7.5$ what are N_1 and N_2?

$N_1 =$ 2.377 $N_2 =$ 3.190

\Y1 = 2.1X/(1+.1X^2) Xmin = 0 Xmax = 15 Zoom Fit

\Y2 = X

2nd calc 1: Value X = 7.5 Y = 2.377

2nd calc 1: value Y1 X = 2.377, Y = 3.190

(b) What are the equilibrium points? *If you are using the calculator to find them* Xmin=0 *and* Xmax=15 *are reasonable choices. Be sure to say how you used the calculator.*

Equilibrium points are: 0, 3.3166

2nd calc: 5: intersect

X = Y = 3.3166

X = 0 is seen to be equilibrium point by the graph

(c) What are the stable equilibrium points? Stable points are: 3.3166

at X = 0, $\frac{dy}{dx}$ = 2.01 > 1 so unstable

X = 3.3166 $\frac{dy}{dx}$ = −.9476 between −1 & 1 stable

(d) If $N_0 = 13$ estimate N_{87}. $N_{87} \approx$ 3.317

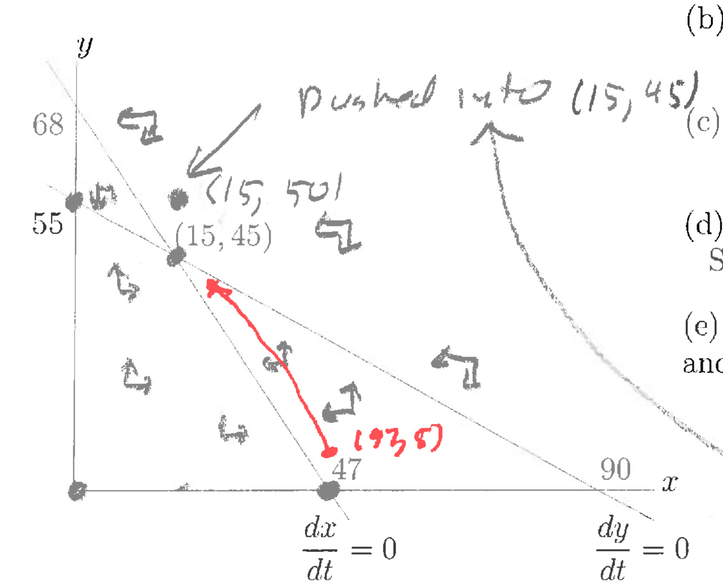
It will cobweb into the stable point.

4. (20 points) The following are phase diagrams for the equations

$$\frac{dx}{dt} = r_1 \left(\frac{K_1 - x - \alpha y}{K_1} \right)$$
$$\frac{dy}{dt} = r_2 \left(\frac{K_2 - \beta x - y}{K_2} \right)$$

of competing species.

(a) Fill in arrows showing in what direction that points are moving in each region.



(b) What is K_1?

$K_1 =$ 47 = carrying capacity.

(c) What are the rest points?

Rest points are: (0,0), (47,0), (0,55), (15,45)

(d) What are the stable rest points?

Stable points are: (15, 45)

(e) If $x(0) = 15$ and $y(0) = 50$ estimate $x(100)$ and $y(100)$.

$x(100) \approx$ 15

$y(100) \approx$ 45

(f) Which of the following describes the long term behavior of this system (circle one).

Completive coexistence ***Completive exclusion***

x-species dominates ***y-species dominates.***

(g) If there is no x-species present, what is the stable y-population size?

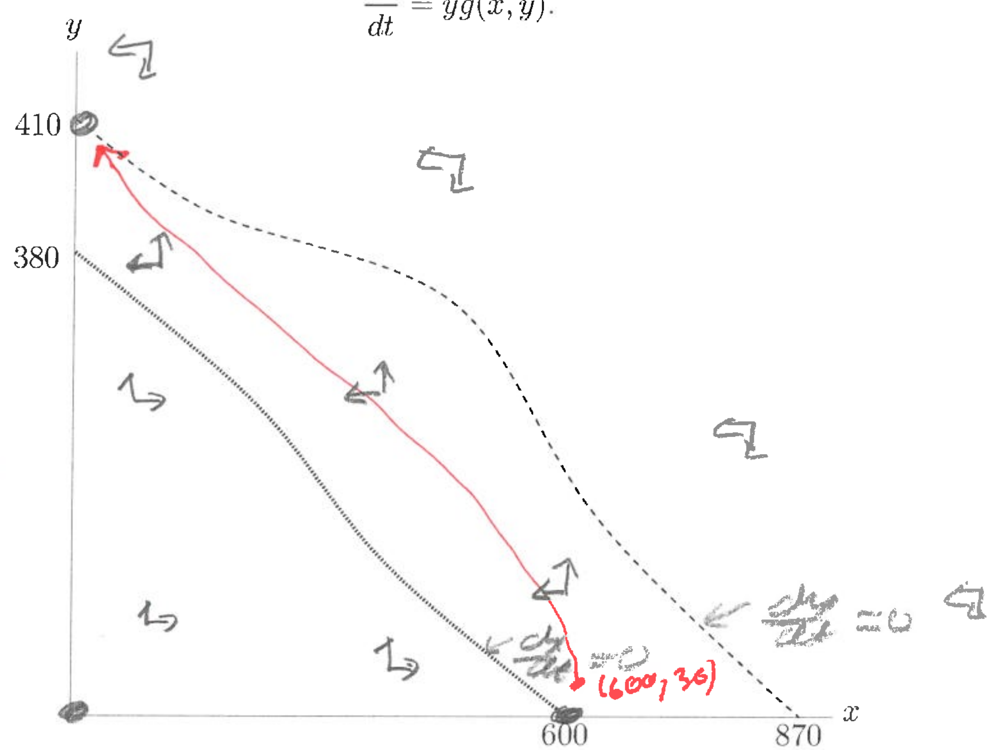
Stable size is 55

(h) Assume that to begin with there are no y-species present and there is a stable x population of size 47. Then 5 of the y-species is introduced. What happens in the long run?

starting at (47, 5) we move up to the stable point (15,45). So in the long run we stabilize at $x=15$, $y=45$.

5. (20 points) The figure below is the phase space for the system

$$\frac{dx}{dt} = xf(x,y)$$
$$\frac{dy}{dt} = yg(x,y).$$



- - - - - - - - - - - - $g(x,y) = 0$ on this curve and is positive below it.

·················· $f(x,y) = 0$ on this curve and is positive below it.

(a) Fill in the arrows showing what direction points are moving in each region.

(b) What are the rest points? Rest points are: (0,0), (600,0), (0,410)

(c) What are the stable rest points Stable points are: (0,410)

(d) If there is no x-species present, what is the carrying capacity for the y-species?

Capacity is 410

(e) If there is no y-species present, what is the carrying capacity for the x-species?

Capacity is 600

(f) If at first there is no y-species present, and the x-species is at its carrying capacity and 30 of the y-species are introduced to the region, then what happens in the long run?

starting at (600,30) the point is pushed up to the stable point (0,410). So the x-species dies off and the y-species stabilizes at y=410