Mathematics 242 Homework.

In doing these problems you will have to some integration by parts. If you need review on this here is a link to the Kahn Academy videos on the subject:
`https://www.khanacademy.org/math/ap-calculus-bc/bc-integration-new/bc-6-11/v/deriving-integration-by-parts-formula`

The Youtube Chanel `https://www.youtube.com/user/patrickJMT` also has many examples of integration by parts:
`https://www.youtube.com/user/patrickJMT/search?query=parts`

Problem 1. Find the general solution to the following differential equations. Note that in some of these problems it may not be possible to solve for the dependent variable in terms of the independent variable.

(a) $y' = 9x^2 + 5x - 3$.

(b) $\dfrac{du}{dt} = \frac{1-t}{3u^2+5}$.

(c) $y' - 3y = x^3$.

(d) $xy' + 4y = \cos(2x)$.

Problem 2. Find the solutions to the following initial value problems.

(a) $y' = x\ln(x), \quad y(1) = 6$.

(b) $\dfrac{dy}{dx} = \dfrac{1-x}{y-2}, \quad y(1) = 5$.

(c) $\dfrac{dP}{dt} - .1P = 2t + 1, \quad P(0) = 100$.

Problem 3. (This problem should be done along with reading Section 1.5.2 on page 47 of the text.) We are experts on solving first order linear equations $y' + p(x)y = f(x)$ by the method of integration factors. There is somewhat more general class of equations that can be reduced to the first order linear equation. Consider the ***Bernoulli equation***

$$y' + p(x)y = f(x)y^n.$$

To try to simplify let $y = v^\alpha$. Then

$$y' = \alpha v^{\alpha-1}v'.$$

(a) Do this substation for y into the Bernoulli equation and show that the result can be simplified to

$$v' + \frac{p(x)}{\alpha}v = \frac{f(x)}{\alpha}v^{\alpha(n-1)+1}.$$

(b) Bernoulli then had the good idea of setting

$$\alpha(n-1) + 1 = 0$$

and solving for α. Show that this gives

$$\alpha = \frac{1}{1-n}.$$

(c) With this choice of α show that the equation for v reduces to

$$v' + (1-n)p(x)v = (1-n)f(x)$$

which is first order linear. We can therefore solve for v. Then $y = v^{\frac{1}{1-n}}$ is a solution to the original differential equation. □

Problem 4. (Optional, consider this problem as extra credit.) Use the method of the last problem (or Section 1.5.2 of the text) to solve the Bernoulli equation

$$y' + (x^2 - 1)y = -xy^6$$

with the initial condition $y(1) = 1$. □

Problem 5. For the Bernoulli equation

$$y' + 2y = 12e^x y^3$$

find both the general solution and the solution with $y(0) = 5$. □

Problem 6. Find both the general solution and the solution with $y(0) = 4$ to the equation

$$y' + 6x^2 y = -12x^2.$$ □

Problem 7. Find the general solution to the equation

$$\frac{dy}{dx} = \frac{y}{x} + 4e^{\frac{3y}{x}}.$$ □

Problem 8. If $k > 0$ is a constant find the solution to the initial value problem

$$y' + ky = x \qquad y(0) = y_0.$$ □

Problem 9. Let a and b be constants. Show that solutions to

$$\frac{dy}{dx} = \frac{-(x-a)}{(y-b)}$$

are parts of circles with center (a, b). □