Mathematics 242 Homework.

Problem 1. Here are some integration practice problems. Most, but not all, them will involve integration by parts. In all of these s is a constant.

(a) $\displaystyle\int e^{-st}t^3\,dt,$

(b) $\displaystyle\int e^{-st}\cos(3t)\,dt$

(c) $\displaystyle\int e^{-st}e^{5t}\,dt$

(d) $\displaystyle\int e^{-st}t^2e^{5t}\,dt$

A pair of functions that will come up are the ***hyperbolic sine and cosine***. These are defined by

$$\cosh(t) = \frac{e^t + e^{-t}}{2}$$
$$\sinh(t) = \frac{e^t - e^{-t}}{2}$$

Problem 2. Here are some basic properties of these function for you to prove.

(a) Their derivatives are

$$\frac{d}{dt}\cosh(t) = \sinh(t)$$
$$\frac{d}{dt}\sinh(t) = \cosh(t).$$

(b) Them satisfy the identity

$$\cosh(t)^2 - \sinh(t)^2 = 1.$$

□

Problem 3. We have seen that the general solution to the equation

$$x''(t) - x(t) = 0$$

is

$$x(t) = c_1e^t = c_2e^{-t}.$$

Show that the functions

$$x_1(t) = \cosh(t), \qquad x_2 = \sinh(t)$$

are also solutions to $x''(t) - x(t) = 0$ and this it is also possible to write the general solution as

$$x = C_1\cosh(t) + C_2\sinh(t).$$

□

Problem 4. Let s be a constant and compute the following integrals:

$$\int e^{-st}\cosh(t)\,dt$$
$$\int e^{-st}\sinh(t)\,dt$$ □

Problem 5. These problems review some properties of improper integrals. Compute the following

(a) $\int_0^\infty t^2 e^{-3t}\,dt$,

(b) $\int_0 e^{2t}e^{-st}\,dt$ where s is a constant with $s > 2$. Why is it important that $s > 2$? □

Problems to be turned on Sunday October 18.

Problem 1. From the exercises on page 299 of the text do the following problems. Probably the easiest way to to use the table of Laplace Transforms on pages 443 and 444 of the text. If you use this table, what you did. For example if the problem was to find the inverse Laplace transform of $\left(\frac{3}{s-2}+\frac{5}{x+6}\right)$ say something to the effect of from the table in the book we see that

$$\mathcal{L}\{e^{-at}\} = \frac{1}{x+a}$$

so the so we have the following inverse transforms

$$\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} = e^{2t}$$
$$\mathcal{L}^{-1}\left\{\frac{1}{s+6}\right\} = e^{-6t}$$

therefore

$$\mathcal{L}^{-1}\left(\frac{3}{s-2}+\frac{5}{x+6}\right) = 3e^{2t}+5e^{-6t}.$$

(a) Exercise 6.1.5
(b) Exercise 6.1.6
(c) Exercise 6.1.9
(d) Exercise 6.1.10 □

Here are two differential equations to solve using Laplace transforms. The basic formulas needed are

$$\mathcal{L}\{x'(t)\} = s\mathcal{L}\{x(t)\} - x(0)$$
$$\mathcal{L}\{x''(t)\} = s^2\mathcal{L}\{x(t)\} - sx(0) - x'(0)$$

Problem 2. Solve the initial value problem

$$x'(t) - 4x(t) = 3e^{2t}, \quad x(0) = 5$$

using Laplace transforms and showing all your steps. □

Problem 3. Solve the initial value problem

$$x''(t) - x'(t) - 6x(t) = 0 \qquad x(0) = 6, \quad x'(0) = 8.$$

using Laplace transforms and showing all your steps. □