Mathematics 546 Homework, October 14, 2020

Problem 1. Let H and K be subgroups of the group G. Show the intersection, $H \cap K$, is also a subgroup. $\square$

Problem 2. Give an example of two subgroups H and K of the dihedral group D_4 such that the union, $H \cap K$, is not a subgroup. $\square$

We have defined the ***alternating group***, A_n, to be the subgroup of S_n consisting of all even permutations.

Problem 3. List the elements of A_3. $\square$

Problem 4. If we number the vertices of a square counterclockwise as 1, 2, 3, and 4. Then the rotation by 90° counterclockwise is represented by the 4-cycle $(1, 2, 3, 4)$. Let b be the reflection in the line through 1 and 3. Then $b = (2, 4)$ then (you do not have to check this) $a^4 = b^2 = 1$ and $ba = a^{-1}b$, which is our usual representation of the dihedral group D_4. List the permutations in $A_4 \cap D_4$. $\square$

Problem 5. Likewise we can represent D_5 as permutations in S_5 with

$$a = (1, 2, 3, 4, 5) \quad \text{and} \quad b = (2, 5)(3, 4).$$

Again it can be checked that $a^5 = 1 = b^2$ and $ba = a{-}1b$ (and again you do not have to check this). List the permutations in $A_5 \cap D_5$. (There is a way to do this with almost no calculation.) $\square$