Mathematics 554 Homework.

For the test corrections to the test, redo the problems you missed (if you only missed part of a problem you one have to do the part you missed). This applies to both the take home and in class part of the test. Turn in both the corrections and the old copy so that I can compare.

Problem 1. This problem is review, but we will be using the result shortly it is worth revisiting. Let A be a closed bounded subset of $\mathbb{R}$. Show that $\sup(A) \in A$. $\square$

Problem 2. More review. Give an example of a bounded subset, A, of $\mathbb{R}$ where $\sup(A) \notin A$. $\square$

Problem 3. Let S be a subset of $\mathbb{R}$ and assume that there are points $a, b \in S$ with $a < b$ and a point $c \notin S$ with $a < c < b$. Show S is disconnected. $\square$

Problem 4. Show that if $S \subseteq \mathbb{R}$ has the property that if $a, b \in S$ with $a < b$, then $[a, b] \subseteq S$, then S is an interval. *Hint:* I have never found a really nice way to do this. Here is one way to start. Let

$$\alpha = \sup(S), \qquad \beta = \inf(S)$$

when these exist. If $\sup(S)$ does not exist, then S is not bounded above so, as short hand, we write $\alpha(S) = +\infty$ in this case. Like wise if $\inf(S)$ does not exist, then S is not bounded below, so we write $\inf(S) = -\infty$.

(a) Show that if $\alpha < x < \beta$, then $x \in S$. (Here, as expected $-\infty < x$ and $x < +\infty$ for all x.

Now things split into cases.

(b) $\alpha = -\infty$ and $\beta = +\infty$, then $S = (-\infty, +\infty) = \mathbb{R}$.
(c) $\alpha = -\infty$ and $\beta < \infty$. Then $S = (-\infty, \beta)$ or $S = (-\infty, \beta]$
(d) $-\infty < \alpha$ and $\beta = +\infty$. Then $S = (\alpha, \infty)$ or $S = [\infty, \infty)$. (As this is just the same as the proof of Part (c) with the inequalities reversed, you are allowed to just write "Similar to Part (c)".)
(e) Both α and β are finite. Then S is one of the four intervals (α, β), $(\alpha, \beta]$, $[\alpha, \beta)$, or $[\alpha, \beta]$. $\square$