Mathematics 554 Homework.

Problem 1. Given an ε-δ proof that
$$\lim_{x \to 5} x^2 + x = 30. \qquad \square$$

The following is a somewhat expanded version of Theorem 4.13 of *Notes on Analysis*.

Theorem 1 (Continuous Functions are Great). *Let $f \colon X \to Y$ be a map between metric spaces. Then the following are equivalent:*

(a) f is continuous
(b) f does the right thing to limits: For all $x_0 \in X$ $\lim_{x \to x_0} f(x) = f(x_0)$.
(c) f does the right thing to limits of convergent sequences in X: If $\lim_{n \to \infty} x_n = x_0$, then $\lim_{n \to \infty} f(x_n) = f(x_0)$.
(d) Preimages of open sets by f are open: If V is an open subset of Y, then $f^{-1}[V]$ is an open subset of X.
(e) Preimages of closed sets by f are closed: If F is a closed subset of Y, then $f^{-1}[F]$ is a closed subset of X. $\qquad \square$

Problem 2. Problem 4.20 on Page 83 of *Notes on Analysis*. $\qquad \square$

Problem 3. Problem 4.24 on Page 85 of *Notes on Analysis*. $\qquad \square$

Problem 4. Problem 4.32 on Page 88 of *Notes on Analysis*. $\qquad \square$

Problem 5. Problem 4.33 on Page 89 of *Notes on Analysis*. $\qquad \square$

Problem 6. Problem 4.34 on Page 89 of *Notes on Analysis*. $\qquad \square$