Mathematics 554 Homework.

I know that some of this overlaps with what we have done in class, but these are definitions and results that you need to get down cold. Due Wednesday after the break do the following from Notes on Analysis.

Problems 3.33 to 3.39

And here are a few more problems.

Problem 1. Let E be a metric space and $\langle p_n \rangle_{n=1}^{\infty}$ a sequence in E which has a limit: $\lim_{n\to\infty} p_n = p$.

(a) Show that we also have $\lim\limits_{n\to\infty} p_{n-5} = p$.

(b) Likewise show $\lim\limits_{n\to\infty} p_{2n} = p$

Problem 2. Let $\lim_{n\to\infty} p_n = p$ in the metric space E. Let $\varepsilon > 0$ and let N be such that $n \geq N$ implies $d(p_n, p) < \varepsilon/2$. Show $m, n \geq N$ implies

$$d(p_m, p_n) < \varepsilon$$

Problem 3. Show that the $x_n = (-1)^n$ has no limit in $\mathbb{R}$. *Hint:* Assume, towards a contradiction, that $\lim_{n\to\infty} x_n = L$. Let $\varepsilon = 1/2$. Then there would exist N such that $n \geq N$ implies $|x_n - L| < \varepsilon = 1/2$. Let $n > N$. Then also $n + 1 > N$. Show $|x_n - x_{n+1}| = 2$ but $|x_n - L| < 1/2$ and $|x_{n+1} - L| < 1/2$ implies $|x_n - x_{n+1}| < 1$ which gives a contradiction.

Problem 4. Let $\langle p_n \rangle_{n=1}^{\infty}$ be a sequence of real numbers such that each p_n is an integer. Show that this sequence converges if and only if it is eventually constant (that is there is a N such that $p_n = p_m$ for all $m, n \geq N$).