Mathematics 554H/703I Test 1 Name: _____

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (5 points) What is the sum of the series $S = \sum_{k=0}^{49} x^2(1-x)^k$? $S =$ _____

2. (10 points) (a) Define the binomial coefficient $\binom{n}{k} =$

(b) State the ***binomial theorem***.

(c) Simplify $\dfrac{(a+h)^3-(a-h)^3}{h}$ (the answer should have no h in the denominator).

$$\frac{(a+h)^3-(a-h)^3}{h} = \text{\_\_\_\_\_}$$

3. (5 points) Give an example of a subset of $\mathbb{R}$ which is bounded below, but which does not have a minimum (no proof needed).

4. (10 points)

(a) Define what if means for a function $f\colon [a,b] \to \mathbb{R}$ to be ***Lipschitz***.

(b) Show the function $f(x) = \dfrac{2x}{x+3}$ is Lipschitz on the interval $[0,4]$.

5. (10 points) Show that

$$x^2 + 2xy + 2y^2 \geq 0$$

with equality if and only if $z = y = 0$.

6. (20 points) (a) Let $S \subseteq \mathbb{R}$ be a nonempty subset of $\mathbb{R}$. Define what it means for S to be ***bounded above.***

(b) Define what it means for b to be a ***least upper bound*** of S.

(c) State the ***least upper bound axiom***.

(d) State ***Archimedes' axiom***.

(e) Let $A \subseteq \mathbb{R}$ be a nonempty set oof positive numbers with the property that if $a \in A$, then also $(1.01)a \in A$. Use the least upper bound axiom to show that A has no upper bound in $\mathbb{R}$.

7. (10 points) Show that if $|x| \geq \max\{1, 2(|a) + |b|)\}$ that

$$1 + \frac{a}{x} + \frac{b}{x^2} \geq \frac{1}{2}.$$