1. (10 points) Define or state the following.

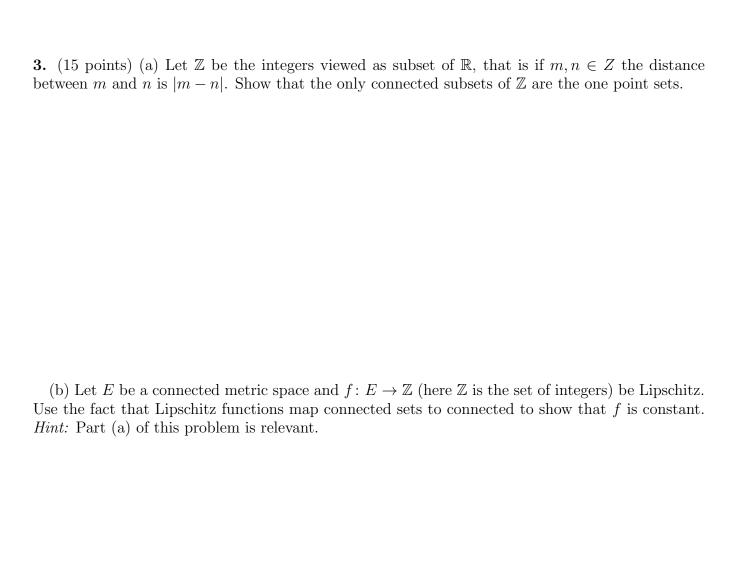
- (a) K is a **compact** subset of the metric space E.
- (b) K is a **sequentially compact** subset of the metric space E.

(c) The metric space E is connected.

(d) A condition on a subset F of the metric space E which is equivalent to F being closed. (More than one answer is possible, just chose one.)

(e) The  $\textbf{\textit{Lipschitz Intermediate Value Theorem}}$  for a Lipschitz function  $f \colon [a,b] \to \mathbb{R}$ .

**2.** (15 points) Use the Intermediate Value Theorem (which we know is true for polynomials) to show that if  $f: [a,b] \to [a,b]$  is a polynomial, then f has a fixed point in [a,b], that is there is an  $x_0 \in [a,b]$  such that  $f(x_0) = x_0$ . Hint: Consider the polynomial g(x) = f(x) - x. What can you say about g(a) and g(b)?



4.	(10 po	ints) Sh	now that	if $K$ is a	compact	subset o	f the met	cric space	E, then $E$	K is close	ed in $E$ .

**5.** (10 points) Let  $K_1 \supseteq K_2 \subseteq K_3 \supseteq K_4 \supseteq \cdots$  be a sequence of nonempty compact subsets of the metric space E. Show the intersection is nonempty. That is

$$\bigcap_{n=1}^{\infty} K_n \neq \varnothing.$$

Hint: One way to do this is show that if this is false and  $U_n = \mathcal{C}(K_n)$ , then  $\{U_n\}_{n=1}^{\infty}$  is an open cover of  $K_1$  and show this leads to a contradiction. (You can use that compact sets are closed.)

<b>6.</b> (5 points) Draw a picture of two connected subset $A$ and $B$ of $\mathbb{R}^2$ such that $A \cap B$ is not connected (no proof required).
7. (5 Points) Which of the letters
a b c d e f c h i j k l m n o p q r s t u v w x y z are disconnected? (Again no proof required.)
The disconnected letters are: