Mathematics 172 Homework.

We have been looking at discrete dynamical systems of the form

$$P_{t+1} = f(P_t).$$

First off there is just the basic problem of computing with these.

Problem 1. For the discrete dynamical system

$$P_{t+1} = P_t + 1.4P_t\left(1 - \frac{P_t}{200}\right) \qquad P_0 = 75$$

Find P_1, P_2, P_3, and P_4.

Solution. Use the calculator to find that

$$P_1 = 140.625$$
$$P_2 = 199.072$$
$$P_3 = 200.365$$
$$P_4 = 199.853$$

□

Problem 2. For the system

$$N_{t+1} = 1.2N_t e^{1-N_t/20} \qquad N_0 = 9$$

find N_1, N_2, N_3, and N_4.

Solution. This time we get

$$N_1 = 18.7191$$
$$N_2 = 23.9486$$
$$N_3 = 23.5895$$
$$N_4 = 23.6567$$

□

Problem 3. For the system of Problem 2 find the equilibrium points by solving the equation

$$N = 1.2Ne^{1-N/20}$$

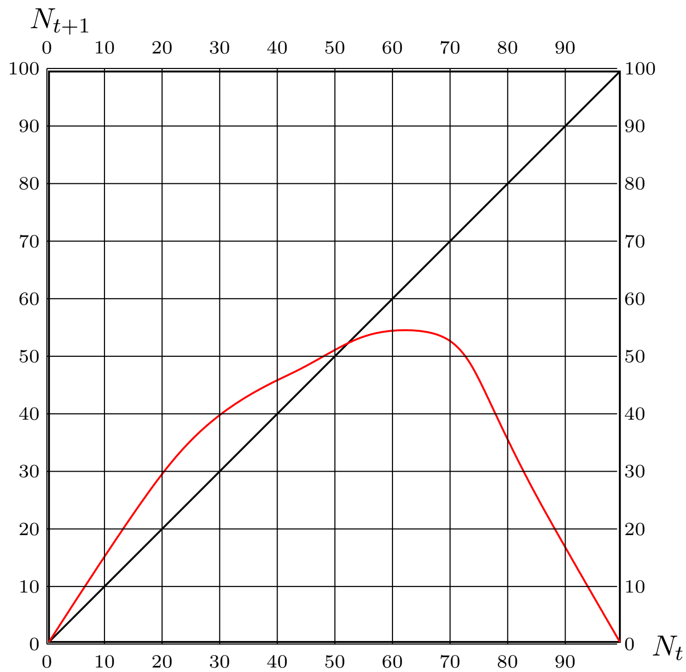
on your calculator.

Solution. One solution is $N = 0$, which we can see just by looking at the equation. To find the other one plot \Y1 = 1.2X(1-X/20) and \Y2 = X with Xmin = 0 and Xmax = 30 and use the calculator to find the ingestion of the graphs. The other equilibrium point is $N = 23.6464$. □

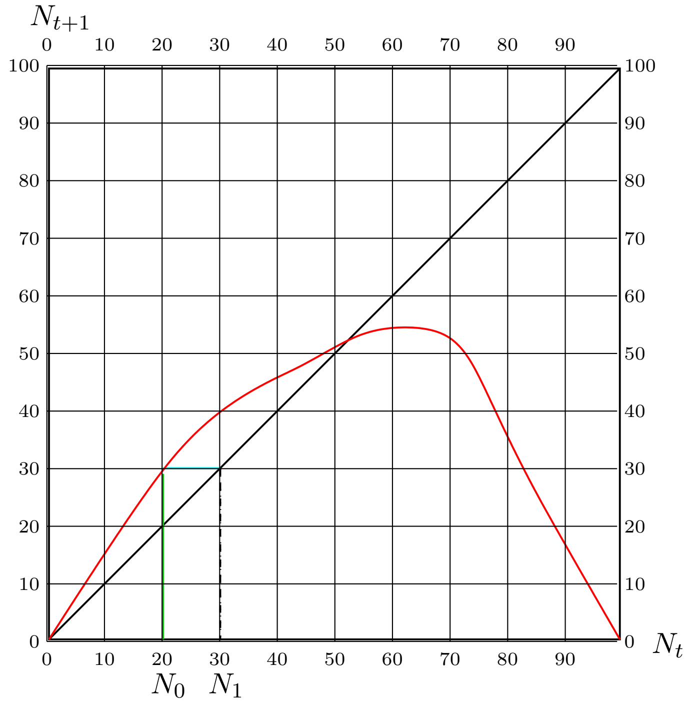
We also want to be able to understand these systems geometrically. We do this by a method usually called ***cobwebbing***. To cobweb a system

$$N_t = f(N_t)$$

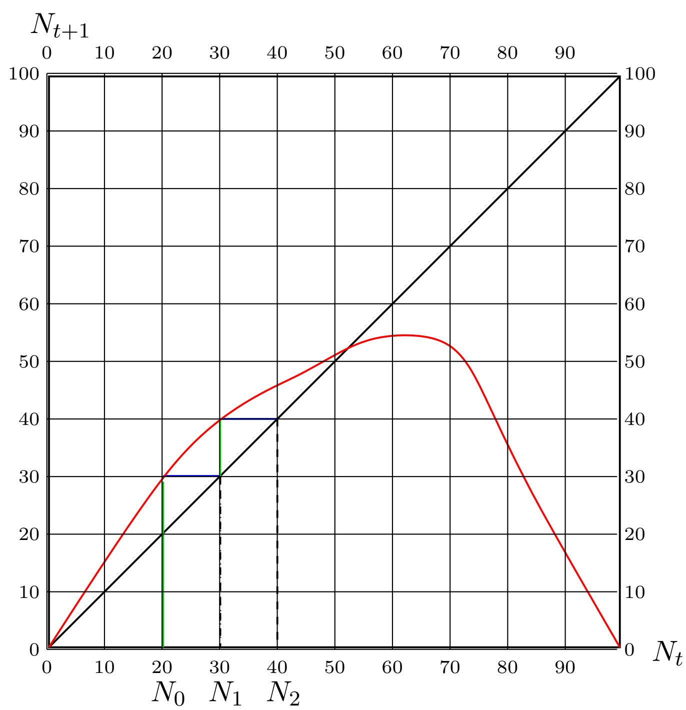
first draw the graph of $N_{n+1} = f(N_t)$ and the graph of the line $N_{t+1} = f(N_t)$ on the same axis:



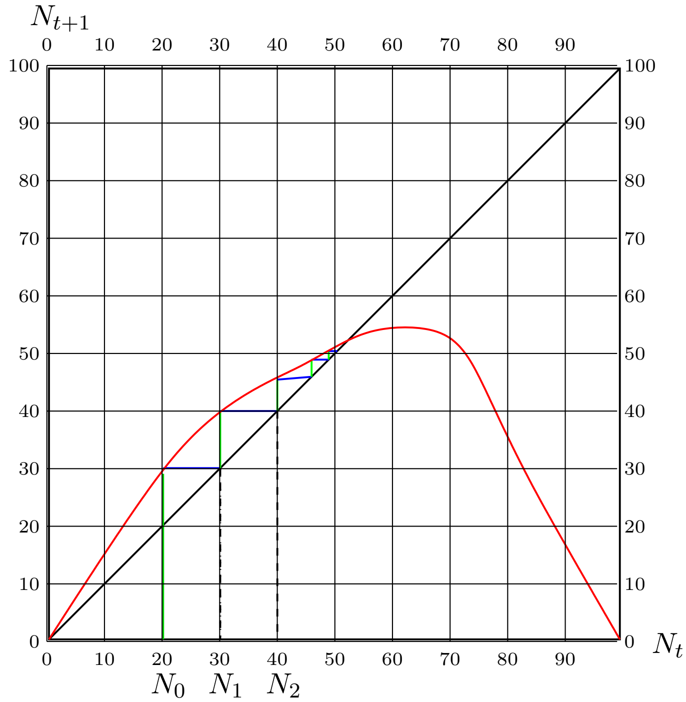
Here the graph of $N_{t+1} = f(N_t)$ is in red and the diagonal line is the graph of $N_{t+1} = N_t$. Here the axis are labeled from 0 to 100 for no reason other than that was easy to do the software I am using. In general the axis with be determined by what values that the N_t's take on. Now let us assume $N_0 = 20$ and estimate N_1. We start with N_0 on the N_t axis and move vertically to the graph. Then horizontally (blue) to the line. Then the x coordinate (really the N_t coordinate) of this point is N_1. You should figure out why this works (hint $N_t = N_{t+1}$ on the diagonal line).



To get N_2 go vertically (green again) to the graph and then horizontally (blue) to the line.

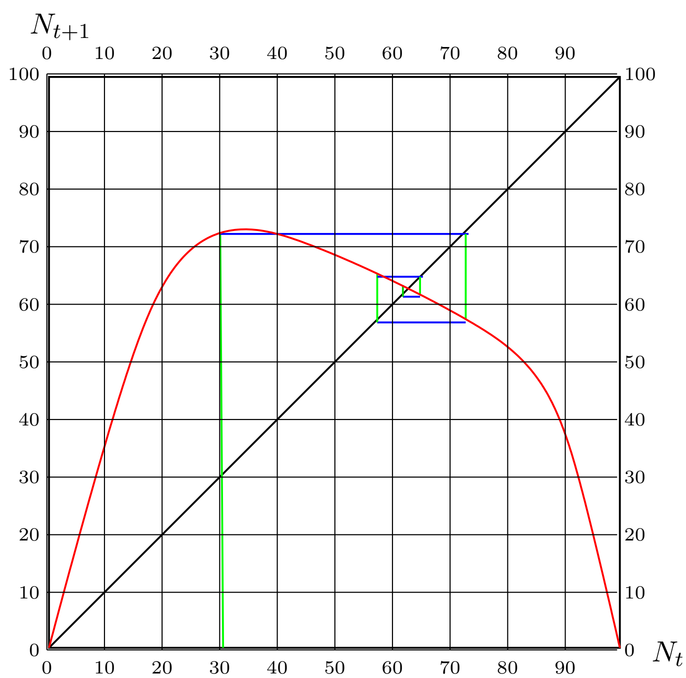


We can keep going like this to get our cobweb (which in this case looks more like a set of steps).



It is clear (I hope) that these steps lead up to the point where the two graphs cross, which looks to be approximately $N = 52$. Thus we can give estimates like $N_{100} \approx 52$, and we did this without having to touch a calculator or computer.

Here is anther example where the picture looks more like a cobweb. This time we are starting with $N_0 = 30$.



Again we end up where the two graphs cross which looks to be about $N = 62$. So in this case if we start with $N_0 = 30$, then we should have $N_{100} \approx 62$.

Here is anther picture where using arrows to show the direction the points are moving. The green trajectory starts at $N_0 = 23$ and after several steps ends up with $N_t \approx 0$. The blue trajectory starts at $N_0 = 30$ and ends up where the graph and line cross between 50 and 60 which looks to be approximately $N = 56$. So if t is large we have $N_t \approx 56$.