Mathematics 172 Homework, August 28, 2023.

Recall that the circle of radius r has

$$\text{Length} = 2\pi r.\text{Area} \qquad = \pi r^2$$

1. What happens if to the length and area of a circle if the radius is doubled?

$\square$

Solution: Let $\mathcal{C}_1$ be a circle of radius r and $\mathcal{C}_2$ a circle of radius $2r$. Let L_1 and A_1 be the length and are of $\mathcal{C}_1$ and L_2 and A_2 the length of $\mathcal{C}_2$. Then

$$\begin{aligned} L_2 &= 2\pi(2r) \\ &= 2(2\pi r) \\ &= 2L_1. \\ A_2 &= \pi(2r)^2 \\ &= 4(\pi r^2) \\ &= 4A_1 \end{aligned}$$

Thus doubling the radius doubles the length, but doubling the radius multiples the area by a factor of 4.

2. What happens if to the length and area of a circle if the radius is tripled?

$\square$

Solution: A calculation which looks almost identical to the one for the previous problem gives

$$L_2 = 3L_1 \qquad A_2 = 3^3 L_1 = 9L_1.$$

$\square$

3. Let $\mathcal{C}_1$ be a circle of radius and λ a positive number. Let $r_2 = \lambda r$. We call λ the ***scaling factor*** or ***magnification factor***. So in Problem 1 we had $\lambda = 2$ and in Problem 2 the value was $\lambda = 3$. Let $\mathcal{C}_2$ be a circle of radius $r_2 = \lambda r$. How are the lengths and areas of the two circles related?

Solution: The calculation is just about the same as the ones we did for $\lambda = 2$ and $\lambda = 3$. The result is

$$L_2 = \lambda L_1, \qquad A_2 = \lambda^2 A_1.$$