Mathematics 172 Homework, November 17, 2023.

This homework is to practice using the calculator to find powers of Leslie matrices to find the number in each stage after t years. To give a fist example let

$$L = \begin{bmatrix} 0 & 2.2 & 25.5 \\ 0.05 & 0 & 0 \\ 0 & 0.7 & 0 \end{bmatrix}$$

and let us start with

$$\vec{N}(0) = \begin{bmatrix} 230 \\ 15 \\ 10 \end{bmatrix}$$

Then after t years the vector

$$\vec{N}(t) = L^t \vec{N}(0)$$

gives the number in each each stage. To do this on the TI-84 we enter the matrix in as the matrix $[A]$ and enter $\vec{N}(0)$ as $[B]$. If you need review on entering the matrices on the calculator the video

`https://www.youtube.com/watch?v=UGccb18KLG0`

does a good job of explaining how this is done. There is also a video for the n-spire and entering matrices.

`https://www.youtube.com/watch?v=5_DVISfgHUo`

Then computing

$$\vec{N}(1) = L\vec{N}(0) = [A][B] = \begin{bmatrix} 288 \\ 11.5 \\ 10.5 \end{bmatrix}$$

For year 2 we have

$$\vec{N}(2) = L^2\vec{N}(0) = [A]^2[B] = \begin{bmatrix} 293.05 \\ 14.4 \\ 8.05 \end{bmatrix}$$

and for year 20 we have

$$\vec{N}(20) = L^{20}\vec{N}(0) = [A]^{20}[B] = \begin{bmatrix} 275.8827115 \\ 14.41878521 \\ 9.192771187 \end{bmatrix}$$

Here are some problems for you to practice on.

1. If

$$L = \begin{bmatrix} 0 & 19.5 & 34 \\ .02 & 0 & 0 \\ 0 & .9 & 0 \end{bmatrix}$$

and

$$\vec{N}(0) = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

show

$$\vec{N}(1) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{N}(2) = \begin{bmatrix} 19.5 \\ 0 \\ .9 \end{bmatrix}, \qquad \vec{N}(50) = \begin{bmatrix} 19.89994328 \\ 0.3976904751 \\ 0.3576465247 \end{bmatrix}$$

2. If

$$L = \begin{bmatrix} 0 & .55 & 6.4 \\ .2 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$$

and

$$\vec{N}(0) = \begin{bmatrix} 213 \\ 52 \\ 35 \end{bmatrix}$$

show

$$\vec{N}(1) = \begin{bmatrix} 252.6 \\ 42.6 \\ 36.4 \end{bmatrix}, \qquad \vec{N}(2) = \begin{bmatrix} 256.39 \\ 50.52 \\ 29.82 \end{bmatrix}, \qquad \vec{N}(100) = \begin{bmatrix} 296.5439719 \\ 59.22226259 \\ 41.34816579 \end{bmatrix}$$