# Mathematics 172 Homework, September 13, 2023.

Let $P(t)$ satisfy a rate eqaution

$$\frac{dP}{dt} = f(P).$$

Then a number $P_*$ such that $f(P_*) = 0$ is an ***equilibrium point***, also called a ***rest points***, or ***stationary point***. Then the constant function $P(t) = P_*$ is a solution to the rate equation. Here is a first example. Let

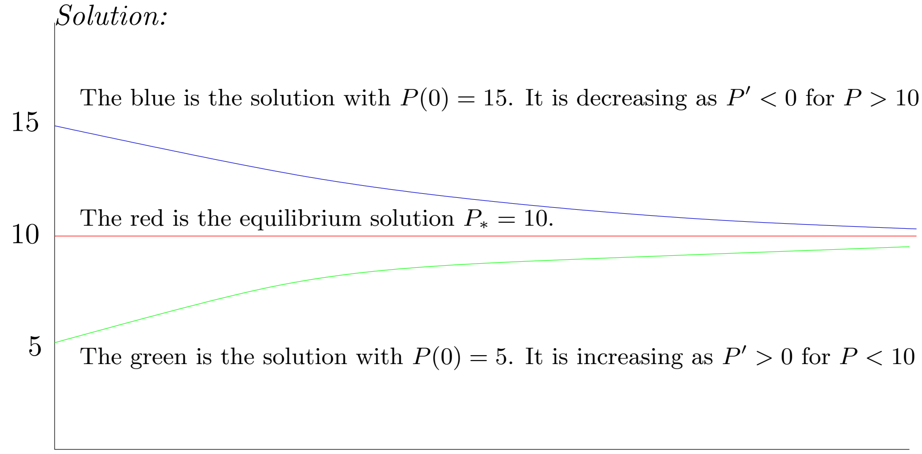
$$P' = 10 - P.$$

**Problem 1.** Find the equilibrium point(s) of this equation.

*Solution:* Solve $10 - P = 0$ to get $P_* = 10$ as the only equilibrium point. □

**Problem 2.** For this same equation note that if $P > 10$, then $P' = 10 - P < 0$. Thus $P$ is a decreasing function when $P > 10$. Likewise if $P < 10$, then $P' = 10 - P > 0$. Use this information to sketch graphs of the solutions with $P(0) = 5$, $P(0) = 10$, and $P(0) = 15$.

*Solution:*




**Problem 3.** In the last problem let for the solutions with $P(0) = 5$ and $P(0) = 15$ estimate $P(100)$.

*Solution:* We see from the graph that both solutions have $P = 10$ as an asymptote. Thus $P(100) \approx 10$ for both solutions. There is nothing special about the number 100 other that is large. For example for both solutions we have $A(123) \approx 10$, $A(1{,}000) \approx 10$, and $A(666) \approx 10$. In the language of calculus the precise statement is $\lim_{t\to\infty} A(t) = 10$. □

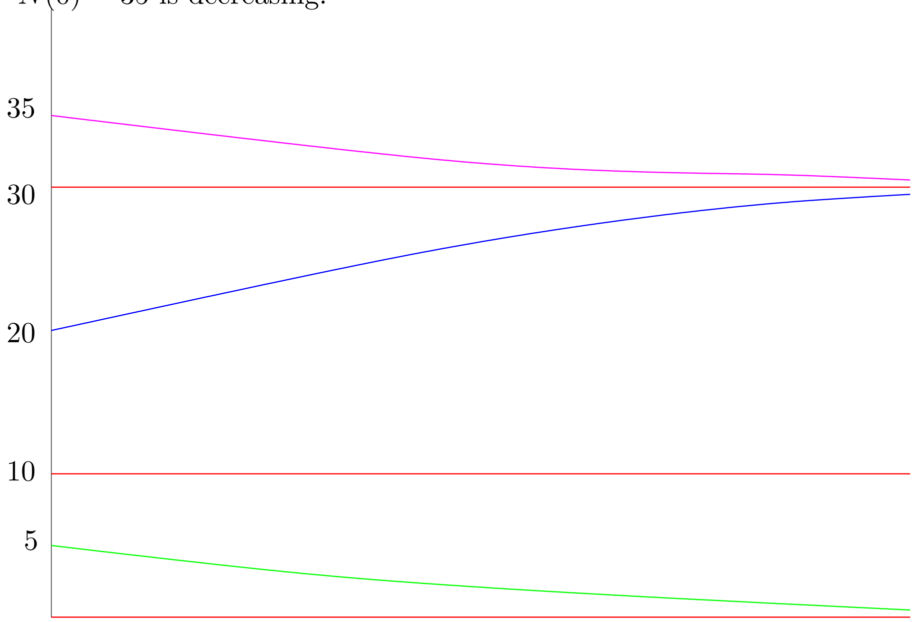
**Problem 4.** For the rate equation

$$N' = .1N(N - 10)(30 - N)$$

(a) Find the equilibrium points. *Solution:* Set $.1N(N - 10)(30 - N) = 0$ and solve to get that the equilibrium points are $N_* = 0, 10, 30$.

(b) Sketch the graphs of the equilibrium solutions together with the solutions that have $N(0) = 5$, $N(0) = 20$, and $N(0) = 35$.

*Solution:* The equilibrium solutions are in red. The solution witn $N(0) = 5$ is in green, the solution with $N(0) = 20$ is in blue, and the solution with $P(0) = 35$ is in magenta. To decide if the solution is increasing or decreasing use the sign of $N'$ (for example $N' < 0$ for $N > 30$ so the solution with $N(0) = 35$ is decreasing.




(c) For the solution with $N(0) = 5$ estimate $N(50)$. *Solution:* $N(50) \approx 0$.

(d) For the solution with $N(0) = 20$ estimate $N(78)$. *Solution:* $N(78) \approx 30$.

(e) For the solution with $N(0) = 35$ estimate $N(200)$. *Solution:* $N(200) \approx 30$.

(f) Which of the equilibrium points are stable? *Solution:* $N_* = 0$ and $N_* = 30$ are stable.

(g) Which of the equilibrium points are unstable? *Solution:* $N_* = 10$ is unstable.