Mathematics 172

Quiz 12 **Name:** Key

You must show your work to get full credit.

1. The acceleration due to gravity is $g = 32$ ft/sec^2. A furlong is 220 yards = 660 feet. A fortnight is two weeks so that

$$1 \text{ fortnight} = 14 \times 24 \times 60 \times 60 = 1{,}209{,}600 \text{ sec.}$$

What is g in furlongs/fortnight^2?

$g = $ 70,939,741,090.9 $\dfrac{\text{furlongs}}{\text{fortnight}^2}$

$$g = 32 \frac{\text{ft}}{\text{sec}^2}$$

$$= 32 \frac{\left(\frac{\text{fur.}}{660}\right)}{\left(\frac{\text{fort.}}{1{,}209{,}600}\right)^2}$$

$$= 32 \frac{(1209600)^2}{660} \frac{\text{fur}}{(\text{fort})^2}$$

2. A large alligator is 13 feet long and weights 790 lbs. It has aa skull that is 1.2 feet long. Deinosuchus was a prehistoric alligator which lived between 82 to 73 million years ago. A partial skeleton of a Deinosuchus is found and its skill is 3.2 feet long. Assume that Deinosuchus had the same proportions as modern alligator and estimate the length and weight of the Deinosuchus

Length ≈ 34.67 ft Weight ≈ 14,986.4 lbs

Using the skull measurements we have the scaling factor is $\lambda = \frac{3.2}{1.2} = 2.667$

Length scales by λ so

Length of Deino. $= 2.667(13) = 34.67$ ft

Weight scales by λ^3

Weight of Deino. $= (2.667)^3 790 = 14986.4$ lbs

3. The crushing pressure of red ceder is 4,560 psi. Assume that a red ceder with a height of 5 feet, the area of its base is .4 feet^2 and weighs 60 lbs. Then what is the critical height where a red ceder crushes itself under its own weight?

Critical height is 152 ft

Scale the tree by a factor of λ.

Scaled weight is $W_\lambda = 60\lambda^3$ lbs

scaled base area is $A_\lambda = .4\lambda^2$.

Average presure on base is

$$\frac{\text{Weight}}{\text{Area}} = \frac{W_\lambda}{A_\lambda} = \frac{60\lambda^3}{.4\lambda^2} = 150\lambda \text{ psi}$$

At critical height $150\lambda = 4{,}560$ so

$$\lambda = \frac{4560}{150} = 30.4.$$

Thus critical hight is $5\lambda = 5(30.4) = 152$ ft

4. A large vat of grape juice has .02 grams of yeast added to it. For the first several hours the size of the yeast colony grows with a constant intrinsic growth rate. After a half hour there is .03 grams of yeast in the vat. Let $W(t)$ be the number of grams of yeast in the vat after t hours.

(a) What is the intrinsic growth rate? $r =$ .811

$W(t) = .02e^{rt}$

$W(.5) = .02e^{.5r} = .03$

$e^{.5r} = \frac{.03}{.02} = 1.5$

$.5r = \ln(1.5)$

$r = \ln(1.5)/.5 = .811$

(b) Give a formula for $W(t)$. $W(t) = .02e^{.811t}$

(c) How long until there is a kilogram (that is 1,000 grams) of yeast in the vat? $t =$ 13.34 hours

Solve

$$.02e^{.811t} = 1000$$

$$e^{.811t} = \frac{1000}{.02}$$

$$t = \ln(1000/.02)/.811$$

$$= 13.34$$

5. A population grows by the rate equation

$$\frac{dP}{dt} = -.12P\left(1-\frac{P}{50}\right)\left(1-\frac{P}{10}\right).$$

(a) If $P(4)=13$ what is $P'(4)$? $P'(4) =$ $.34632$

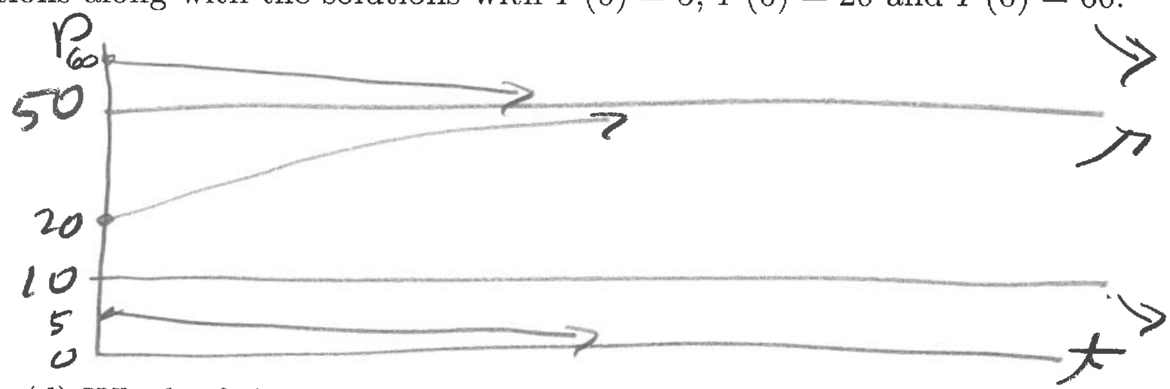
$P'(4) = -.12(13)(1-\frac{13}{50})(1-\frac{13}{10}) = .34632$

(b) What are the equilibrium points?

The equilibrium points are: $0, 10, 50$

Solve $-.12P(1-\frac{P}{50})(1-\frac{P}{10}) = 0$

(c) Make a graph of the solutions to the equation showing the equilibrium solutions along with the solutions with $P(0)=5$, $P(0)=20$ and $P(0)=60$.



(d) Which of the equilibrium points are stable?

The stable points are: $0, 50$

(e) If $P(0)=9$ estimate $P(98)$. $P(98) \approx$ 0

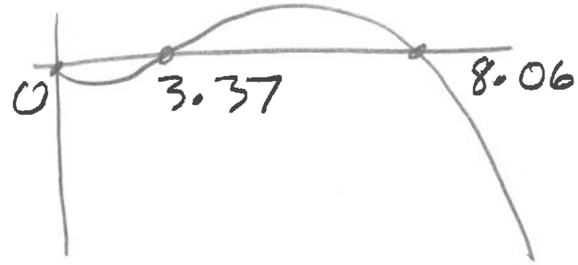
This solution will decrease down to 0. So for any large t $P(t) \approx 0$. Thus $P(98) \approx 0$

6. For the rate equation $\frac{dP}{dt} = -.07P^3 + .8P^2 - 1.9P$

(a) Use your calculator to make a graph of $\frac{dP}{dt}$ as a function of P and sketch the graph here. *Hint:* Use Xmin=0 and Xmax=10.

\Y1 = -.07X³ + .8X² - 1.9X

Zoom 0: ZoomFit



(b) What are the equilibrium points? Give your answer to two decimal places.

P=0 is clear. Equilibrium points are: 0, 3.37, 8.06

For the other two use 2nd calc 2:Zero

(c) What are the stable equilibrium points?

Stable points are where slope is negative, i-e down hill. Stable points are: 0, 8.06

7. A population of brewers yeast in a large tank grows logistically with a intrinsic growth rate of $r = .25$ lbs/hour and a carrying capacity of $K = 600$. Let $P(t)$ be the number of pounds of yeast in the tank after t hours.

(a) Write the rate equation satisfied by P. *Remark:* A rate equation is an equation (so there is an equal sign in it) and also contains a rate (that is a derivative).

Logistic equation is

$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right)$

The rate equation is $\frac{dP}{dt} = .25P\left(1-\frac{P}{600}\right)$

(b) One the yeast has is at its carrying capacity, it is harvested at a constant rate of 10lbs/hour. Write the new rate equation satisfied by P.

The new equation is $\frac{dP}{dt} = .25P\left(1-\frac{P}{600}\right) - 10$

(c) What is the new stable size of the yeast population in the tank?

Plot $\frac{dP}{dt}$ as function of P. The stable size is 556.90

\Y1 = .25X (1 - X/600) - 10

Xmin = 0

Xmax = 600

Zoom 0: ZoomFit

Stable point

2nd calc 2:Zero