Mathematics 172

Quiz 14 **Name:** Key

You must show your work to get full credit.

1. A maker of health foods is raising a type of micro algae to sell as a nutritional supplement. The population of algae in a tank is growing logistically with $r = .9$ (lbs/day)lb and a carrying capacity of $K = 25$ lbs. What is the maximum rate that the algae can be harvested without kill off the population? Write a sentence or two, or draw a picture, explaining how you got your answer.

The equation is $\frac{dP}{dt} = .9P(1-\frac{P}{25})$

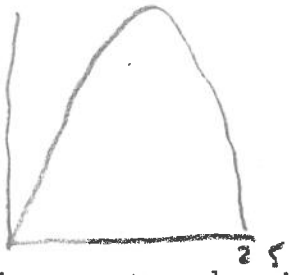
Plot $\frac{dP}{dt}$ as function of P

$Y1 = .9X(1-X/25)$

Xmin = 0

Xmax = 25

Maximum harvest at is 5.625 lbs/day



Max of $\frac{dP}{dt}$ is 5.625 which is maximum harvesting rate

2. A population of plants is a pasture has its population modeled by the Allee equation

$$\frac{dP}{dt} = -.4P\left(1-\frac{P}{20}\right)\left(1-\frac{P}{75}\right)$$

where $P(t)$ is the size of the population after t weeks.

(a) If cattle are introduced to the pasture that eat the plant at constant rate of 6 plants/week, what is the new stable population size of the plants. Write a sentence or two, or draw a picture, explaining how you got your answer.

The equation is $\frac{dP}{dt} = -.4P(1-\frac{P}{20})(1-\frac{P}{75}) - 6$

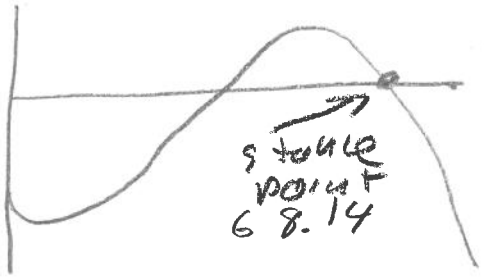
Plot $\frac{dP}{dt}$ as function of P

$Y1 = -.4X(1-X/20)(1-X/75)-6$

Xmin = 0

Xmax = 75

Stable population size is 68.14 ≈ 69 plants

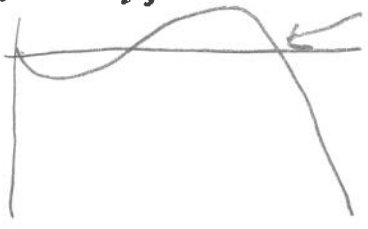


(b) If instead of the cattle being introduced to the pasture assume that a virus kills off 15% of the plants each week. What is the new stable population size? Write a sentence or two, or draw a picture, explaining how you got your answer.

This time $\frac{dP}{dt} = -.4P(1-\frac{P}{20})(1-\frac{P}{75}) - .15P$

Graph

Stable population size is 61.41 plants ≈ 61 plants



stable point 61.41