

# Mathematics 172

## Quiz 19

Name: Key

*You must show your work to get full credit.*

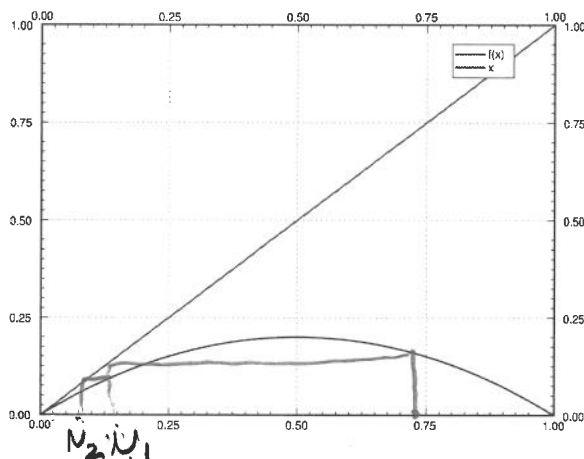
1. If  $N_0 = .7$  estimate

$$N_1 \approx \underline{.25}$$

$$N_2 \approx \underline{.12}$$

$$N_{45} \approx \underline{0}$$

by drawing in the cobwebs.

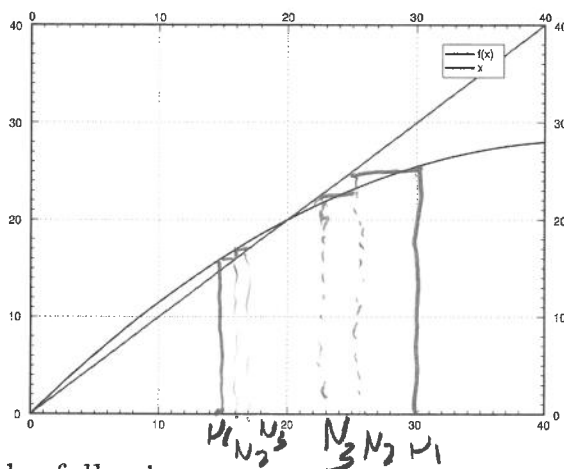


What are the equilibrium points and which are stable?

Equilibrium points 0

Stable points 0

2. The graph here is for the discrete logistic system with per capita growth rate  $r = .3$  and carrying capacity  $K = 20$ .



If  $N_0 = 30$  estimate the following

$$N_1 \approx \underline{25}$$

$$N_2 \approx \underline{23}$$

$$N_{32} \approx \underline{30}$$

If  $N_0 = 15$  estimate the following

$$N_1 \approx \underline{16}$$

$$N_2 \approx \underline{17}$$

$$N_{45} \approx \underline{30}$$

What are the equilibrium points and which are stable?

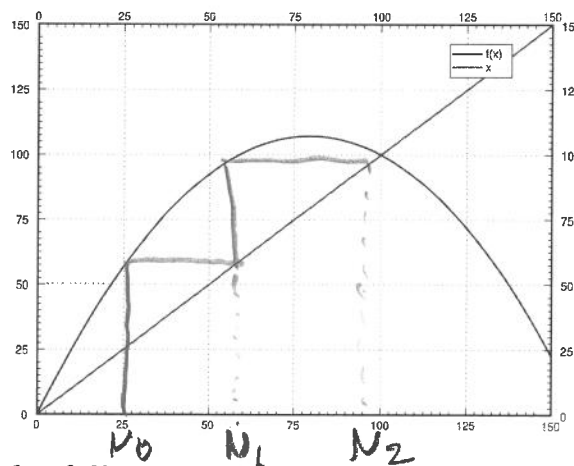
Equilibrium points 0, 30

Stable points 30

If  $0 < N_0 < 40$  what can you say about  $N_{100}$ ? Write a sentence explaining your answer.

$N_{100} \approx 30$  as  $N_0, N_1, N_2, \dots$  will converge to the stable equilibrium point

3. The graph here is for the discrete logistic system with per capita growth rate  $r = 1.7$  and carrying capacity  $K = 100$ .



If  $N_0 = 25$  estimate the following

$N_1 \approx$  56

$N_2 \approx$  95

$N_{83} \approx$  100

What are the equilibrium points and which are stable?

Equilibrium points 0, 100

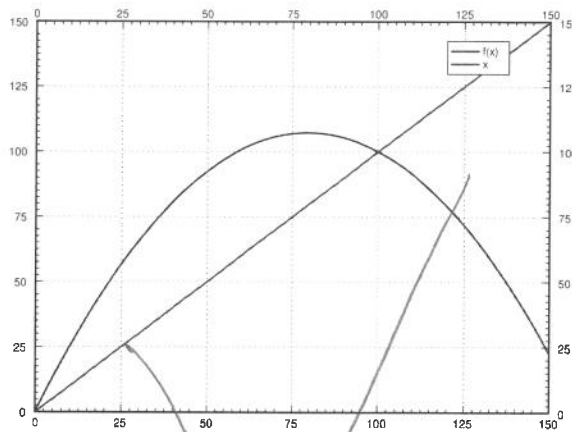
Stable points 100

Write a sentence or two describing the long term behaviour this system.

Equilibrium points 0, 100 Stable points 100

4. The graph here is for the discrete logistic system with per capita growth rate  $r = 1.7$  and carrying capacity  $K = 100$ .

Repeated by accident



If  $N_0 = 25$  estimate the following

$$N_1 \approx \underline{\hspace{2cm}}$$

$$N_2 \approx \underline{\hspace{2cm}}$$

$$N_{83} \approx \underline{\hspace{2cm}}$$

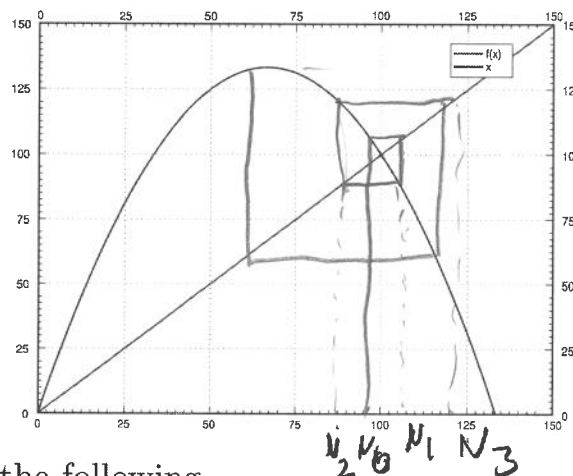
What are the equilibrium points and which are stable?

Equilibrium points 0, 100

Stable points 0, 100

Write a sentence or two describing the long term behaviour this system.

5. The graph here is for the discrete logistic system with per capita growth rate 2.5 and carrying capacity  $K = 100$ .



If  $N_0 = 95$  estimate the following

$$N_1 \approx \underline{110}$$

$$N_2 \approx \underline{90}$$

$$N_3 \approx \underline{12}$$

What are the equilibrium points and which are stable?

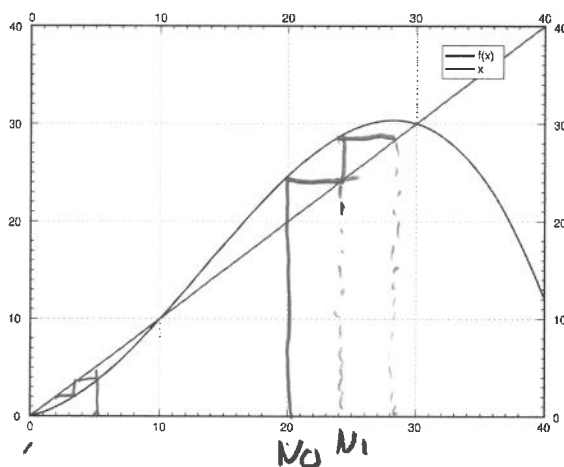
Equilibrium points 0, 100

Stable points None

If  $N_0 = 95$  can you make a reasonable estimate of  $N_{10}$ ? If not why is this case different from the previous two examples? Write a few sentences explaining your answer.

NO. There are no stable equilibrium points, so  $N_1, N_2, \dots$  does not tend to any limit.

6. Here is the graph of a system  $N_{t+1} = f(N_t)$



If  $N_0 = 20$  estimate

$$N_1 \approx 24$$

$$N_2 \approx 28$$

$$N_{32} \approx 30$$

If  $N_0 = 5$  estimate

$$N_1 \approx 4$$

$$N_2 \approx 2.5$$

$$N_{128} \approx 0$$

What are the equilibrium points and which are stable?

Equilibrium points 0, 10, 30 Stable points 0, 30 Write a few sentences explaining the long term behaviour of this system.

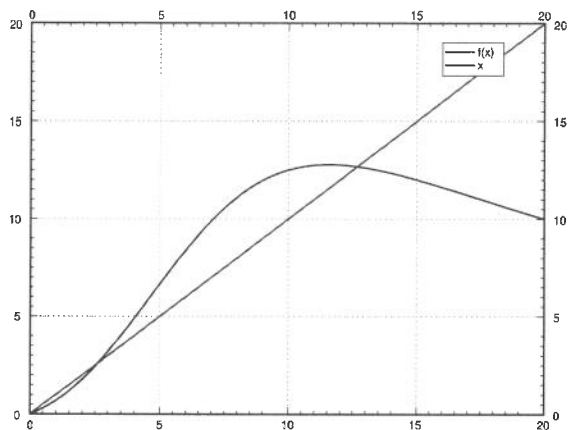
7. The following is the graph of  $N_{t+1}$  as a function of  $N_t$  for the system

If  $N_0$  is between 0 and 10, then  $N_t$  goes to the stable point 0

If  $N_0$  is between 10 and 40 it goes to the stable point 30

so in long run, depending on the value of  $N_0$  either the population dies or stabilizes at  $N=30$

$$N_{t+1} = \frac{.5N_t + .2N_t^2}{1 + .001N_t^3}$$



Use your calculator to find the equilibrium points.

The equilibrium points are 0, 2.587, 12.67

Use your calculator to find  $dy/dx$  at the equilibrium points. (List the points and the derivatives).

$$\frac{dy}{dx} = 0.5$$

0

$$\frac{dy}{dx} = 1.45$$

2.58

$$\frac{dy}{dx} = -1.18$$

12.67

Which of the points are stable?

Stable points are 0, 12.67

Write a few sentences explaining the long term behaviour of this system.

If  $N_0$  is between 0 and 2.58  
the population dies off.

If  $N_0$  is between 2.58 and  
20 it stabilizes at 12.67.