

Mathematics 172

Quiz 21

Name: Key

You must show your work to get full credit.

Recall the basic formula for using the derivative to do approximations:

$$f(a+h) \approx f(a) + f'(a)h.$$

1. IP $P(3) = 7.2$ and $P'(3) = 1.5$ approximate $P(3.1)$ and $P(3.05)$

$$P(3.1) \approx \underline{7.35}$$

$$a=3, h=.1$$

$$P(3.1) \approx P(3) + P'(3)(.1) \\ = 7.2 + (1.5)(.1) = 7.35$$

$$P(3.05) \approx \underline{7.275}$$

$$P(3.05) \approx 7.2 + (1.5)(.05) \\ = 7.275$$

2. Let P satisfy the rate equation

$$P'(t) = .12P(t) \left(1 - \frac{P(t)}{115} \right)$$

- (a) If $P(4) = 105$ compute $P'(4)$.

$$P'(4) = \underline{1.096}$$

$$P'(4) = .12(105) \left(1 - \frac{105}{115} \right) \\ = 1.096$$

- (b) Use this to approximate $P(4.05)$.

$$P(4.05) \approx \underline{105.055}$$

$$P(4.05) \approx P(4) + P'(4)(.05) \\ = 105 + (1.096)(.05) \\ = 105.055$$

- (c) If $P(13) = 118$ compute $P'(13)$ and use this to approximate $P(13.1)$.

$$P'(13) = \underline{-0.3694}$$

$$P(13.1) \approx \underline{117.963}$$

$$P'(13) = .12(118) \left(1 - \frac{118}{115} \right) = -0.3694$$

$$P(13.1) \approx P(13) + P'(13)(.1) \\ = 118 + (-0.3694)(.1) \\ = 117.963$$

We can use the same idea on systems of rate equations. For example let us work with the system

$$\begin{aligned}\frac{dx}{dt} &= .1x(1 - .1x - .2y) \\ \frac{dy}{dt} &= .2y(1 - .15x - .15y)\end{aligned}$$

Assume

$$x(17) = 5, \quad y(17) = 2$$

3. Compute the derivatives $x'(17)$ and $y'(17)$.

$$x'(17) = \underline{.05} \quad y'(17) = \underline{-.2}$$

$$x'(17) = .1(5)(1 - .1(5) - .2(2)) = .05$$

$$y'(17) = .2(2)(1 - .15(5) - .15(2)) = -.001$$

4. Use your answer to the previous problem to approximate $x(17.5)$ and $y(17.5)$.

$$x(17.5) \approx \underline{5.025} \quad y(17.5) \approx \underline{1.995}$$

$$\begin{aligned}x(17.5) &\approx x(17) + x'(17)(.5) \\ &= 5 + .05(.5) \\ &= 5.025\end{aligned}$$

$$\begin{aligned}y(17.5) &\approx y(17) + y'(17)(.5) \\ &= 2 + (-.001)(.5) \\ &= 1.995\end{aligned}$$