Mathematics 172

Quiz 25 **Name:** Key

You must show your work to get full credit.

This is to clarify something that caused some confusion on Test 2. We have two notions of unconditional growth: discrete and continuous.

For discrete growth we have a population N_1 where $t = 0, 1, 2, 3, \ldots$. If r is the pre capita growth rate, then N_t satisfies

$$N_{t+1} = N_t + rN_t = (1 + r)N_t.$$

If $\lambda = 1 + r$, then this can also be written as

$$N_{t+1} = \lambda N_t$$

and the solution is

$$N_t = N_0\lambda^t.$$

For the continuous version of unconstrained growth we have a population size $P(t)$ depending on $t \geq 0$ (sometimes this is called a continuous variable) and if r is the intrinsic growth rate (note this is not the same as the per capita growth rate) then this satisfies the rate equation (= differential equation)

$$\frac{dP}{dt} = rP$$

which has the solution

$$P(t) = P_0e^{rt}.$$

1. If a population is modeled with a unconstrained discrete population growth and $N_0 = 10$ and $N_4 = 25$

(a) What is the equation satisfied by N_t?

The equation is $N_{t+1} = (1+r)N_t$ or $N_{t+1} = \lambda N_t$

(b) Give a formula for N_t. $N_t =$ $10(1.2574)$

$N_t = 10\lambda^t$ $\quad 4 = 25$ $\quad \lambda^4 = 25/10$ $\quad \lambda = (25/10)^{\frac{1}{4}} = 1.2574$

(c) What is the per capita growth rate? $r =$ $.2574$

$r = \lambda - 1 = .2574$

(d) What is N_{10}? $N_{10} =$ 98.795

$N_{10} = 10(1.2574)^{10}$

$=$

2. If a population is modeled with a unconstrained continuous model and $P(0) = 10$ and $P(4) = 25$

(a) What is the rate equation satisfied by $P(t)$?

The equation is $P' = rP$ or $\frac{dP}{dt} = rP$

(b) Find a formula for $P(t)$. $P(t) = 10e^{.2291t}$

$P(t) = 10e^{rt}$

$P(4) = 10e^{4r} = 25$

$e^{4r} = \frac{25}{10}$

$4r = \ln(25/10)$

$r = \ln(25/10) = .2291$

(c) What is the intrinsic growth rate? $r = .2291$

(d) What is $P(10)$?

$P(10) = 10e^{.2291(10)}$

$= 98.85$

If had used more decimal places this would be true same as 1(a)

(e) What is the per capita growth rate of this population?

The rate is $.2575$

The population at $t=1$ is

$P(1) = 10e^{.2291(1)} = 12.575$

The change in population is

$\Delta P = P(1) - P(0)$

$= 12.575 - 10 = 2.575$

so the per capita change is

$\frac{\Delta P}{P} = \frac{2.575}{10} = .2575$

So this is (up to 3 decimal places) the same as 1(c).